B.Sc. Part—I Semester—II (Old) Examination

MATHEMATICS

(Vector Analysis & Solid Geometry)

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory. Attempt it once only.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :

(1) Volumes of parallelepiped with $\bar{a}, \bar{b}, \bar{c}$ as edge vector is :

(a) $\bar{a} \times (\bar{b} \times \bar{c})$ (b) $\bar{a} \cdot (\bar{b} \times \bar{c})$

(c) $(\bar{a} \times \bar{b}) \times \bar{c}$ (d) $(\bar{a} \times \bar{b}) \times \bar{c}$

(2) The derivative of any constant vector function is :

(a) One (b) Two

(c) Zero (d) None of these

(3) The plane which passes through $p(\bar{r})$ and contains binormal and tangent is said to be :

(a) Osculating plane (b) Rectifying plane

(c) Normal plane (d) None of these

(4) For any space curve $\bar{t}' \bar{b}' =$ ______

(a) K (b) τ

(c) $K\tau$ (d) $-K\tau$

(5) A vector $\bar{f}$ is said to be solenoidal if :

(a) $\text{div } \bar{f} = 0$ (b) $\text{curl } \bar{f} = 0$

(c) $\text{grad } \bar{f} = 0$ (d) $\nabla \cdot \nabla \bar{f} = 0$

(6) If $\text{curl } \bar{f} = 0$ then a vector $\bar{f}$ is :

(a) Rotational (b) Irrotational

(c) Solenoidal (d) None

(7) A plane section of a sphere is :

(a) Circle (b) Point

(c) Line (d) Plane

(8) Any second degree equation in which co-efficient of x^2, y^2, z^2 are equal and product terms xy, yz and zx are absent represents a :

(a) Cone (b) Cylinder

(c) Sphere (d) Plane

(9) Every homogeneous equation of second degree in x, y, and z represents a _____ whose vertex is at origin.

(a) Sphere (b) Cylinder

(c) Cone (d) None

(10) A cone whose generator makes a constant angle with a fixed line through its vertex is called the :

(a) Right circular cylinder (b) Right circular cone

(c) Cylinder (d) None 1×10=10

UNIT—I

2. (a) Prove that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b}) \cdot \bar{c}$.

(b) Show that $\bar{a} \times (\bar{b} \times \bar{c}), \bar{b} \times (\bar{c} \times \bar{a}), \bar{c} \times (\bar{a} \times \bar{b})$ are coplanar. 10

3. (p) Show that the necessary and sufficient condition for $\bar{f}(t)$ to have constant direction is $\bar{f} \times \frac{d\bar{f}}{dt} = 0$.

(q) If $\bar{a} = t\bar{i} - 3\bar{j} + 2t\bar{k}$, $\bar{b} = \bar{i} - 2\bar{j} + 2\bar{k}$ and $\bar{c} = 3\bar{i} + t\bar{j} - \bar{k}$ evaluate $\int_1^2 \bar{a} \cdot (\bar{b} + \bar{c})\, dt$. 10

UNIT—II

4. (a) State and prove Serret-Frenet formulae.

(b) Show that the tangent at any point on the curve whose equation are $x = 3u$, $y = 3u^2$, $z = 2u^3$ makes a constant angle with the line $y = z - x = 0$. 6+4

5. (p) Prove that for any curve

$$[\bar{t}', \bar{t}'', \bar{t}'''] = [\bar{r}'', \bar{r}''', \bar{r}''''] = K^3(K\tau - K'\tau) = K^5(\tau/K)'.$$

(q) Show that necessary and sufficient condition that a curve be a straight line is $K = 0$. 10

UNIT—III

6. (a) Prove that $\bar{r}^n \bar{r}$ is irrotational. Find the value of n when it is solenoidal.

(b) If $\bar{f} = x^2 z\bar{i} - 2y^3 z^2 \bar{j} + xy^2 z\bar{k}$ find div $\bar{f}$ and curl $\bar{f}$ at (1, –1, 1). 10

7. (p) Evaluate $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$ where C is the square formed by the line $y = \pm 1$ and $x = \pm 1$.

(q) Find the work done in moving a particle once around a circle C in the xy plane of radius Z and centre (0, 0) and if force field is given by

$$\bar{f} = 3xy\bar{i} - y\bar{j} + 2zx\bar{k}$$ 10

UNIT—IV

8. (a) Find the equation of the sphere which passes through the point (1, 0,0), (0, 1, 0) and (0, 0, 1) and radius as small as possible.

(b) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3). 10

9. (p) Find the equation of the two sphere which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch $4x + 3y - 15 = 0$.

(q) Two sphere of radii r_1 and r_2 cut orthogonally prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1 + r_2}}$. 10

UNIT—V

10. (a) Find the equation of the cone whose vertex is at the point (α, β, γ) and whose generator touch the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Find the equation of the cone whose vertex is origin and which passes through the curve of intersection of the plane $\ell x + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$. 10

11. (p) Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$.

(q) Find the equation of the right circular cylinder of radius 4 whose axis passes through the origin and makes equal angles with the co-ordinate axes. 10