AE-5099

B.Sc. Part–II (Semester–III) (CBCS) Examination

DSC–VI

MATHEMATICS

Partial Differential Equation

Time : Three Hours] [Maximum Marks : 60

Note :– (1) Question No. **1** is compulsory. Solve it in **ONE** attempt only.

(2) Attempt one question from each unit.

1. Choose the correct alternative : 1 × 10 = 10

(i) $z = \left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)\left(x^n + y^n\right)$ is homogeneous function of degree :

(a) $\frac{n}{2}$ (b) $n + \frac{1}{2}$

(c) $\frac{n+1}{2}$ (d) n^2

(ii) If $z = \log\left(x^2 + xy + y^2\right)$ then value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is :

(a) 1 (b) $\frac{2x+y}{x^2+xy+y^2}$

(c) 2 (d) $\frac{x+2y}{x^2+xy+y^2}$

(iii) The auxiliary equation for the PDE, $P_p + Q_q = R$ is :

(a) $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{R}$ (b) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

(c) $\frac{dx}{R} = \frac{dy}{P} = \frac{dz}{Q}$ (d) None of these

(iv) The singular integral of $q^2 = z^2p^2\left(1 - p^2\right)$ is :

(a) z = 1 (b) z = 2

(c) z = 0 (d) z = 4

(v) The non-linear partial differential equation f (x, y, z, p, q) = 0 and g (x, y, z, p, q) = 0 are compatible iff :

(a) $J_{xp} + J_{yq} + pJ_{zp} + qJ_{zq} = 0$ (b) $J_{xp} + J_{yq} = 0$

(c) $J_{zp} = J_{zq}$ (d) None of these

(vi) The complete solution of $q = 3p^2$ is :

(a) $z = x^2 + 3a^2y + b$ (b) $z = ax + a^2y + b$

(c) $z = ax^2 + 3a^2y - b$ (d) $z = ax + 3a^2y + b$

(vii) The distance ρ between the curves $y = x$ and $y_1 = x^2$ on [0, 2] is :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 2 (d) 4

(viii) The Euler's equation for a functional of the form $\int_a^b f(x, y)dx$ is :

(a) $F_{y'} = C_1$ (b) $F_y - y'F_{y'} = C_1$

(c) $F_y = C_1$ (d) None of these

(ix) The partial differential equation, $\frac{\partial u}{\partial t} = 5\frac{\partial u}{\partial x}$ has a solution u (x, t) is :

(a) $C_5 e^{\lambda\left(t+\frac{x}{5}\right)}$ (b) $C_5 e^{\lambda\left(t-\frac{x}{5}\right)}$

(c) $C_5 e^{\lambda(t+5x)}$ (d) $C_5 e^{\lambda\left(t+\frac{t}{5}\right)}$

(x) In general solution to the given partial differntial equation $u(x,t) = c_5 e^{\lambda\left(t+\frac{x}{12}\right)}$ and $u(x,0) = 6\ e^{\frac{1}{6}x}$, then values of C_5 and λ are :

(a) $\frac{1}{2}$ and 6 (b) 2 and $\frac{1}{2}$

(c) 6 and 2 (d) 6 and $\frac{1}{2}$

UNIT–I

2. (a) If z = f (x, y) be a homogeneous function of degree 'n' then prove that,

(i) $x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial x \partial y} = (n-1)\frac{\partial z}{\partial x}$

(ii) $x\frac{\partial^2 z}{\partial x \partial y} + y\frac{\partial^2 z}{\partial y^2} = (n-1)\frac{\partial z}{\partial y}$

(iii) $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$ 6

OR

(b) Let $z = \sin^{-1}\left(\dfrac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{6}} + y^{\frac{1}{6}}}\right)$, prove that $x^2 \dfrac{\partial^2 z}{\partial x^2} + 2xy \dfrac{\partial^2 z}{\partial x \partial y} + y^2 \dfrac{\partial^2 z}{\partial y^2} = \dfrac{\tan z}{144}\left(\tan^2 z - 11\right)$ 6

(c) Prove that a polynomial function is a homogeneous function of degree n if all of its terms of the same degree n. 4

OR

(d) If z (x + y) = $x^2 + y^2$, show that $\left(\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}\right)$. 4

UNIT–II

3. (a) Find the general integral of P.D.E. $\left(y^3 x - 2x^4\right)p + \left(2y^4 - x^3 y\right)q = 9z\left(x^3 - y^3\right)$. 6

OR

(b) Find the complete integral of P.D.E. $z\left(p^2 - q^2\right) = x - y$. 6

(c) Obtain the singular solution of P.D.E. $z^2\left(1 + p^2 + q^2\right) = k^2$. 4

OR

(d) Solve : $p + 3q = 5z + \tan\left(y - 3x\right)$. 4

UNIT–III

4. (a) Solve : $xyr + x^2 s - yp = x^3 e^y$, where $r = \dfrac{\partial p}{\partial x}, s = \dfrac{\partial p}{\partial y}$. 6

OR

(b) Solve : $\left(D^2 + DD' - 6D'^2\right)z = \cos\left(2x + y\right)$. 6

(c) Solve : $\left(D^2 - 2DD' + D'^2\right)z = \tan\left(y + x\right)$. 4

OR

(d) Prove that the equation f (x, y, p, q) = 0 and g (x, y, p, q) = 0 are compatible if $J_{xp} + J_{yq} = 0$. 4

UNIT–IV

5. (a) Define closedness of curves in the sense of m^{th} order of proximity.

Also show that the curves $y(x) = \dfrac{\sin x}{n}$, where n is sufficiently large and $y_1(x) = 0$ on $[0, \pi]$ are closed in sense of proximity of any order. 1 + 5

OR

(b) Find the shortest distance between two points A and B in terms of polar co-ordinates. 6

(c) For the functional $I[y] = \int_0^1 \left(x^3 y'^2 - y^2\right)dx$, put $y = x^2$, $\delta_y = kx$. Find $\Delta I[y(x)]$ for k = 2. 4

OR

(d) Find the distance between the curves $y(x) = xe^{-x}$ and $y_1(x) = 0$ on [0, 2]. 4

UNIT–V

6. (a) Solve partial differential equation $\frac{\partial u}{\partial t} = 3\frac{\partial u}{\partial x}$ and u (x, 0) = 10 $e^{\frac{1}{6}x}$. 6

OR

(b) Solve partial differential equation, $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ with x (x, 0) = 6 e^{-3x} using method of separable variable. 6

(c) Formulate two separate ODE's for the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ (where u (x, t) represent temperature distribution at position x and time t), describes the diffusion of heat in solid medium over time. 4

OR

(d) Formulate two separate ODE's, for the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (where u (x, t) represent displacement of the wave at position x and time t and c is wave speed) describes the propagation of waves in a medium. 4