$1 \times 10 = 10$

B. Sc. Part-II (Semester-IV) Examination

MATHEMATICS

(Modern Algebra : Groups and Rings)

Paper-VII

[Maximum Marks: 60

Note : $-$ (1)	Question No. 1	is compulsory and	l attempt at once only.

- (2) Solve **one** question from each unit.
- 1. Choose the correct alternatives :

Time : Three Hours]

- (i) A non-empty subset H of the group G is a subgroup of G if and only if $a, b \in H \Rightarrow$
 - (a) $(ab)^{-1} \in H$ (b) $ab^{-1} \in H$
 - (c) $a^{-1}b^{-1} \in H$ (d) None of these
- (ii) The product of an even and odd permutation is :
 - (a) Odd (b) Even
 - (c) Both odd and even (d) None of these
- (iii) If G is a finite group and N is a normal subgroup of G, then O(G/N) is equal to :
 - (a) O(G).O(N) (b) O(G) + O(N)
 - (c) O(G)/O(N) (d) O(G) O(N)
- (iv) A group having only improper normal subgroup is called :
 - (a) A permutation group (b) A finite group
 - (c) A simple group (d) None of these
- (v) If ϕ be a homomorphism of group G onto G' with kernel K, then G' is :
 - (a) Isomorphic to G/K (b) Isomorphic to K/G
 - Isomorphic to G (d) One-one homomorphism
- (vi) The Kernel of a homomorphism $f: G \to G'$ is :
 - (a) A normal subgroup of G (b) A subgroup of G'
 - (c) A normal subgroup of G' (d)
- (vii) A commutative ring without zero divisor is :
 - (a) Boolean Ring (b) Field
 - (c) Division Ring (d) None of these

(c)

1

None of these

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(r) Show that every cyclic group is abelian.		(q)	If G is a group N is a normal subgro operation of multiplication of coset	oup of (ts.	G, then show that G/N is also a group under	the 4				
		(r)	Show that every cyclic group is abeli	an.	311	2				

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UNIT-III

- 6. (a) If ϕ is a homomorphism of a group G into a group G' then prove that :
 - (i) $\phi(e) = e'$
 - (ii) $\phi(\mathbf{x}^{-1}) = (\phi(\mathbf{x}))^{-1} \forall \mathbf{x} \in G$ 4

where e and e' are the unit elements of G and G' respectively.

- (b) Let G be any group, g a fixed element in G. Define $\phi: G \to G$ by $\phi(x) = gxg^{-1}$. Prove that ϕ is an isomorphism of G onto G.
- (c) Show that any kernel is nonempty.
- 7. (p) If φ is a homomorphism of G into G' with kernel K, then prove that K is a normal subgroup of G.
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 - (q) Let G be the group of non-zero real numbers under addition and $G'=\{1,-1\}'$ where 1,1=1,

1.
$$(-1) = (-1)(1) = -1, (-1)(-1) = 1$$
. Define $\phi: G \to G'$

Such that

$$\phi(x) = \begin{cases} 1, & x \text{ is positive} \\ -1, & x \text{ is negative} \end{cases}$$

Show that ϕ is a homomorphism.

(r) Show that the mapping $f: C \to R$ defined by f(x+iy) = x is a homomorphism of the additive group of complex numbers onto the additive group of real numbers and find the kernel of f. 4

UNIT-IV

- 8. (a) If R is a ring in which $x^3 = x$, $\forall x \in R$, then prove that R is a commutative ring. 4
 - (b) Prove that the intersection of two subrings is a subring.
 - (c) Let the characteristics of the ring R be 2 and let $ab = ba \forall a, b \in R$. Then show that $(a + b)^2 = a^2 + b^2$ 3
- 9. (p) Define commutative ring. If R is a ring with zero element O, then for all a, b, c ∈ R, prove that :
 - (i) $a \cdot O = O \cdot a = O$
 - (ii) a(-b)=(-a). b=-(ab)
 - (iii) (-a)(-b) = ab
 - (iv) a.(b-c) = a.b-a.c
 - (q) Let R be a ring with a unit element 1, in which $(ab)^2 = a^2b^2 \forall a, b \in \mathbb{R}$. Prove that R must be commutative. 5

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UNIT-V

- 10. (a) Prove that a homomorphism f of a ring R to a ring R' is an isomorphism iff $K_{eff} = \{0\}$. 4
 - (b) Let R be a commutative ring with unity. Prove that every maximal ideal of R is a prime ideal.
 - (c) If U is an ideal of a ring R with unity 1 and $1 \in U$, then prove that U = R. 3
- 11. (p) Let R and \overline{R} be rings with zero elements O, \overline{O} respectiblely and $f: R \to \overline{R}$ be a homomorphism. Then prove that : 5

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- (i) $f(O) = \overline{O}$
- (ii) $f(-a) = -f(a) \forall a \in R$
- (iii) $f(a-b) = f(a) f(b) \forall a, b \in \mathbb{R}$.
- (q) If F is a field, then prove that its only ideals are $\{0\}$ and F itselt.
- (r) Define:
 - (i) Left ideal
 - (ii) Simple Ring.

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