AD–1897

B.Sc. Part—III Semester—VI Examination
MATHEMATICS
(Special Theory of Relativity)
Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :

(1) The transformations

$$x' = \alpha(x - vt),\ y' = y,\ z' = z,\ t' = \alpha\left(t - \frac{v}{c^2}\right)x,\ \text{where}\ \ \alpha = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

are called as :

(a) Galilean transformations (b) Laplace transformations

(c) Lorentz transformations (d) Fourier transformations

(2) Lorentz transformations reduces to Galilean transformations if :

(a) $v = c$ (b) $v << c$

(c) $v >> c$ (d) None of these

(3) The four dimensional vector $A^r = (A^1, A^2, A^3, A^4)$ (or four vector) is space like if :

(a) $A^2 < 0$ (b) $A^2 > 0$

(c) $A^2 = 0$ (d) None of these

(Here A is the length of the four vector A^r).

(4) The number of distinct components of a symmetric tensor of order 2 in an N-dimensional space are :

(a) N (b) $\frac{N(N-1)}{2}$

(c) $\frac{N(N+1)}{2}$ (d) None of these

(5) If $\overline{A}$ is a vector potential then the magnetic field $\overline{H}$ is :

(a) $\overline{H} = \text{div.}\overline{A}$ (b) $\overline{H} = \text{curl}\ \overline{A}$

(c) $\overline{H} = \text{div.}\ (\text{curl}\,\overline{A})$ (d) None of these

(6) The Electric and Magnetic field strengths $\overline{E}$ and $\overline{H}$ are invariant under :

(a) Galilean transformations (b) Laplace transformations

(c) Lorentz transformations (d) Guage transformations

(7) Four velocity of a particle is :

(a) A unit space-like vector (b) A unit time-like vector

(c) A unit light-like vector (d) None of these

(8) The interval $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + (dx^4)^2$ where $ds^2 > 0$ is said to be :

(a) Time like (b) Space like

(c) Light like (d) None of these

(9) s and s′ are two inertial systems, where s′ is moving with uniform velocity v along xx′ axis the Lorentz contraction factor is :

(a) $\alpha = \sqrt{1 - \frac{v^2}{c^2}}$ (b) $\alpha = -\sqrt{1 - \frac{v^2}{c^2}}$

(c) $\alpha = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ (d) $\alpha = \frac{-1}{\sqrt{1 - \frac{v^2}{c^2}}}$

(10) Kronecker delta is δ^i_j and defined as :

(a) $\delta^i_j = \begin{cases} 1, & \text{for } i \neq j \\ 0, & \text{for } i = j \end{cases}$ (b) $\delta^i_j = \begin{cases} 1, & \text{for } i = j \\ 0, & \text{for } i \neq j \end{cases}$

(c) $\delta^i_j = \begin{cases} 1, & \text{for } i = j \\ -1, & \text{for } i \neq j \end{cases}$ (d) $\delta^i_j = \begin{cases} 1, & \text{for } i \neq j \\ -1, & \text{for } i = j \end{cases}$ 1×10=10

UNIT—I

2. (a) Prove that in an inertial frame a body, without influence of any forces, moves in a straight line with constant velocity. 4

(b) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 3

(c) Show that simultaneity is relative in special relativity. 3

3. (p) Show that Lorentz transformations form a group with respect to multiplication. 4

(q) Show that the circle $x'^2 + y'^2 = a^2$ in s′ is measured to be an ellipse in s if s′ moves with uniform velocity relative to s. 4

(r) What are the postulates of special relativity ? 2

UNIT—II

4. (a) Obtain the transformations for the velocity of a particle under special Lorentz transformations. 5

(b) If $\bar{u}$ and $\bar{u}'$ be the velocities of a particle in two inertial systems s and s′ respectively then prove that :

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{u'^2}{c^2}}\,\sqrt{1-\frac{v^2}{c^2}}}{\left(1+\frac{u'_x v}{c^2}\right)},$$

where s′ is moving with velocity v relative to s along xx′ axis. 5

5. (p) What do you mean by :

(i) Time Dilation

(ii) Length Contraction ? 3+3

(q) In a system s, a particle has a velocity 0.5 c in the xy-plane at an angle 30° with the x-axis. Determine the velocity of the particle in the system s′ which is moving with the velocity 0.4 c relative to s. 4

UNIT—III

6. (a) Define four dimensional radius vector. If the length of four radius vector is denoted by s then show that s^2 is invariant under Lorentz transformations. 1+3

(b) Prove that :

(i) $T'^{11} = \alpha^2\left\{T^{11} - \frac{v}{c}T^{14} - \frac{v}{c}T^{41} + \frac{v^2}{c^2}T^{44}\right\}$

(ii) $T'^{14} = \alpha^2\left\{-\frac{v}{c}T^{11} + T^{14} + \frac{v^2}{c^2}T^{41} - \frac{v}{c}T^{44}\right\}$ 3+3

7. (p) Show that :

$x^1 = -x_1,\ x^2 = -x_2,\ x^3 = -x_3,\ x^4 = x_4$ and then $x_i = (-\bar{r}, ct)$. 4

(q) Prove that there exists an inertial system s′ in which the two events occur at one and the same point if the interval between two events is time-like. 4

(r) Define :

(i) World point

(ii) World line. 2

UNIT—IV

8. (a) Define Four Velocity. Prove that the four velocity in component form is expressed as :

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1 - u^2/c^2}}, \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \right)$$

where $\bar{u} = (u_x, u_y, u_z)$ = velocity of the particle. 1+3

(b) Show that four velocity and four acceleration are mutually orthogonal. 3

(c) Prove that the square of the magnitude of the four-momentum vector p^i is mo^2c^2. 3

9. (p) Define Four Force. Prove that the four force in component form is expressed as :

$$f^i = \left(\frac{\bar{f}}{c\sqrt{1 - u^2/c^2}}, \frac{\bar{f} \cdot \bar{u}}{c^2\sqrt{1 - u^2/c^2}} \right)$$ 1+4

(q) Obtain Einstein's mass-energy equivalence relation. 5

UNIT—V

10. (a) Define Electric field $\overline{E}$ and Magnetic field $\overline{H}$ in terms of scalar potential ϕ and vector potential $\overline{A}$. Also write them in component form. 1+1+3

(b) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is

$$H = \left\{ m_0^2 c^4 + c^2 \left(P - \frac{e}{c} A \right)^2 \right\}^{1/2} + e\phi$$ 5

11. (p) If $\overline{E}$ and $\overline{H}$ are electric and magnetic field strengths then prove that :

(i) $\overline{E} \cdot \overline{H}$ is invariant

(ii) $E^2 - H^2$ is invariant

under the Lorentz transformations. 3+3

(q) Define Current four vector. Show that $c^2\rho^2 - J^2$ is invariant and its value is $\rho_0^2 c^2$. 1+3