B.Sc. Part–III (Semester–VI) Examination

MATHEMATICS

(Special Theory of Relativity)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt only once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :

(1) Sum of two tensors $\overset{\cdot}{A}{}^{ij}_{k}$ and $\overset{\cdot}{B}{}^{ij}_{k}$ is a mixed tensor of order _____.

(a) 6 (b) 9

(c) 3 (d) None of these

(2) The reference system is said to be an inertial system if _____.

(a) Newton's first law of motion valid

(b) Newton's second law of motion valid

(c) Newton's third law of motion valid

(d) None of these

(3) The order of outer product is the _____ of the order of the tensors.

(a) Sum (b) Different

(c) Product (d) None of these

(4) Newton's fundamental equations of motion are invariant under _____.

(a) Lorentz transformation (b) Galilean transformation

(c) General Lorentz transformation (d) None of these

(5) The simultaneity in special relativity is :

(a) Absolute (b) Constant

(c) Relative (d) None of these

(6) In relativistic addition law for velocities when $c \to \infty$. Then _____.

(a) $u' = v - u$ (b) $u' = u - v$

(c) $u' = u + v$ (d) None of these

(7) Four force f^i = _____.

(a) $\frac{dp^i}{ds}$ (b) $\frac{du^i}{ds}$

(c) $\frac{dx^i}{ds}$ (d) None of these

(8) The mass of a moving particle $m = \frac{m_o}{\sqrt{1-\frac{u^2}{c^2}}}$ is called _____.

(a) Equivalent mass of a particle (b) Relativistic mass of a particle

(c) Rest mass of a particle (d) None of these

(9) If $\overline{A}$ is a vector potential then the magnetic field is given by :

(a) $\overline{H} = \text{div } \overline{A}$ (b) $\overline{H} = \text{curl } \overline{A}$

(c) $\overline{H} = \Delta\phi \times A$ (d) None of these

(10) The Maxwell tensor F_{ij} is _____.

(a) $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ (b) $\frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}$

(c) $\frac{\partial A_i}{\partial x^j} + \frac{\partial A_j}{\partial x^i}$ (d) None of these $10\times1=10$

UNIT—I

2. (a) Show that the electromagnetic wave equation :

$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 f}{\partial f^2} = 0$ is not invariant under GT. 4

(b) Show that simultaneity is relative in SR. 3

(c) Show that $x^2 + y^2 + z^2 - c^2t^2$ is Lorentz invariant. 3

3. (a) Prove that Newton's fundamental equations of motion are invariant under the Galilean transformation. 4

(b) Discuss the Geometrical interpretation of Lorentz transformation. 4

(c) Prove that the four dimensional volume element dxdydzdt is Lorentz invariant. 2

UNIT—II

4. (a) Obtain the expression a_x', a_y' and a_z' for acceleration of a particle. 6

   (b) Write short note on Time dilation. 4

5. (a) In the system S', let $u_x' = c \cos \theta$, $u_y' = c \sin \theta$. If S' moves with velocity v relative to the system S along the x-axis then show that $ux^2 + u^2y = c^2$ in S. 4

   (b) Obtain the transformation of Lorentz contraction factor. 6

UNIT—III

6. (a) Define four vectors. Show that the square of the length of a four vector is invariant under LT. 5

   (b) Define space like interval. Prove that there exist an inertial system S' in which the two events occure at one and the same time if the interval between two events is space like. 5

7. (a) Obtain the transformation of the components of a symmetrical four tensor T'^{12}, T'^{23}, T'^{14} under the Lorentz transformation. 6

   (b) Define :

   (i) Four dimensional radius vector

   (ii) World line

   (iii) Light like interval

   (iv) Contravariant tensor of order two. 4

UNIT—IV

8. (a) Deduce Einstein's mass energy equivalence relation. 6

   (b) Show that the quantity $p^2 - \frac{E^2}{c^2}$ is an invariant whose numerical value is — $m_o^2 c^2$. 4

9. (a) Define four force.

   Show that the four force can be expressed as :

$$f^i = \left(\frac{\bar{F}}{c\sqrt{1-\frac{u^2}{c^2}}}, \frac{\bar{F}.\bar{u}}{c^2\sqrt{1-\frac{u^2}{c^2}}} \right)$$

   where $\bar{F} = \frac{d\bar{p}}{dt}$. 5

(b) Derive the relativistic equation :

$$m = \alpha\left(1 + \frac{v\, u_x'}{c^2}\right) m' \quad \text{for mass,}$$

where $\alpha = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$. 5

UNIT—V

10. (a) Show that the Lorentz force acting on a particle of charge 'e' is given by

$$\bar{F}_L = e\left(\bar{E} + \frac{1}{c}\bar{u} \times \bar{H}\right).$$ 6

(b) Describe the field in inertial frame S' if an electromagnetic field is purely electric in an inertial frame S. 4

11. (a) Obtain the transformation for electric and magnetic field strengths. 6

(b) Show that the quantities $\bar{E}$ and $\bar{H}$ remains invariant under Guage transformation. 4