B.Sc. Part–III (Semester–V) Examination

MATHEMATICS

(Mathematical Analysis)

Paper–IX

Time : Three Hours] [Maximum Marks : 60

317

Note :— (1) Q. No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternatives :—

(1) Consider P = (1, 2, 4) be the partition of the interval [1, 4] then μ(P) is :

(a) 1 (b) 0

(c) 2 (d) 4

(2) A bounded function f is Riemann integrable on [a, b] if its :

(a) Upper and lower sum are equal (b) Upper and lower sum are not equal

(c) L(P, f) = –U(P, f) (d) None

(3) The value of B(3, ½) is :

(a) $\frac{15}{16}$ (b) $\frac{16}{15}$

(c) $\frac{11}{15}$ (d) $\frac{15}{11}$

(4) The value of Π is :

(a) 0 (b) 1

(c) –1 (d) n

(5) C–R equations of an analytic function w = u + iv are :

(a) $u_x = v_y$, $u_y = v_x$ (b) $u_x = v_x$, $u_y = v_y$

(c) $u_x = v_y$, $u_y = -v_x$ (d) $u_x = -v_y$, $u_y = v_x$

(6) A function F(x, y) is harmonic in D if :

(a) $F_{xx} + F_{yy} = 0$ (b) $F_{xx} - F_{yy} = 0$

(c) $F_{xy} + F_{yx} = 0$ (d) None

(7) The fixed points of the bilinear transformation $w = \frac{(2+i)z-2}{i+z}$ are :

(a) $\pm i$ (b) $1 \pm i$

(c) 0, –1 (d) 0, 1

(8) The normal form of a bilinear transformation having only one finite fixed point α is :

(a) $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + k$ (b) $\frac{1}{w+\alpha} = \frac{1}{z+\alpha} + k$

(c) $\frac{1}{w-\alpha} = \frac{1}{z-\alpha}$ (d) $\frac{1}{w+\alpha} = \frac{1}{z+\alpha}$

(9) An arbitrary intersection of closed set is :

(a) Closed (b) Open

(c) Neither closed nor open (d) None

(10) If $f : X \to Y$ is continuous mapping and X is compact then :

(a) f(X) is connected (b) $f(X) = \phi$

(c) $F(X) \neq \phi$ (d) f(X) is compact 10×1=10

317 317 317 317 317

UNIT—I

2. (a) If $f \in R[a, b]$ then prove that $F : [a, b] \to R$ defined by

$$F(x) = \int_a^x f(t)\, dt$$

is continuous on [a, b] and if f is continuous at $x_0 \in [a, b]$ then prove that F is differentiable at x_0 with $F'(x_0) = f(x_0)$. 5

(b) Prove that $\int_1^{-1} \cos^8 x\, dx < 0$. 5

3. (p) Prove that every continuous function is integrable. 5

(q) Let the function f be defined as

$$f(x) = \begin{cases} 1 & , \quad x \text{ is rational} \\ -1 & , \quad x \text{ is irrational} \end{cases}$$

show that f is not R-integrable over [0, 1] but $|f| \in R[0, 1]$. 5

UNIT—II

4. (a) Prove that $\beta(m, n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma m+n}$. 4

(b) Evaluate $\int_0^1 x^3 (1-x)^3 \, dx$. 3

(c) Prove that $\Gamma(1/2) = \sqrt{\pi}$. 3

5. (p) Prove that $\int_{a+}^{b} \dfrac{dx}{(x-a)^p}$ converges if $p < 1$ and diverges if $p \geq 1$. 4

(q) Prove that $\int_2^{\infty} \dfrac{x^2 dx}{\sqrt{x^7+1}} < \infty$. 3

(r) Test the convergence of $\int_1^{\infty} \dfrac{\log x}{x+a} dx$. 3

UNIT—III

6. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin although C–R equations are satisfied at the point. 5

(b) Show that the function given by $u = 2x(1 - y)$ is harmonic find v such that $u + iv$ is analytic also find f(z) in terms of z. 5

7. (p) If f(z) and $f(\overline{z})$ are analytic functions then prove that f(z) is constant. 5

(q) Prove that in polar form C–R equations can be written as :

$$\frac{\partial u}{\partial \gamma} = \frac{1}{\gamma}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial \gamma} = -\frac{1}{\gamma}\frac{\partial u}{\partial \theta}.$$ 5

UNIT—IV

8. (a) Consider the transformation $w = ze^{i\frac{\pi}{4}}$. Determine the region in w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane. 5

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. 5

9. (p) Prove that every bilinear transformation with two finite fixed points α, β is of the form

$$\frac{w-\alpha}{w-\beta}=k\frac{z-\alpha}{z-\beta}$$ where k is constant. 5

(q) Find the fixed points of the bilinear transformation $w=\frac{z}{2-z}$. Find its normal form. Show that it is hyperbolic. 5

UNIT—V

10. (a) Let the mapping d : C[0, 1] × C[0, 1] → R be defined by

$$d(f, g)=\int_0^1 |f(x)-g(x)|\,dx$$

Show that d is a metric on C[0, 1]. 5

(b) Prove that every neighbourhood is an open set. 5

11. (p) Let X be a nonempty set and let d be a real function of order pairs of elements of X which satisfies the following condition

$d(x, y) = 0 \Leftrightarrow x = y$ and $d(x, y) \leq d(x, z) + d(y, z)$.

Show that d is a metric on X. 5

(q) Let (X, d) be a metric space and $A \subseteq X$. Prove that A is closed iff A contains its boundary. 5

317 317 317 317 317