AD–1896

B.Sc. Part—III Semester—VI Examination
MATHEMATICS (Graph Theory)
Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt it at once only.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternative in the following :

(1) For Graph G with e edges and n vertices, the sum of degrees of all vertices is equal to :

(a) 2e (b) e/2

(c) e + 1 (d) e – 1

(2) A graph that has neither loops nor parallel edges is called :

(a) Planar graph (b) Complete graph

(c) Simple graph (d) None of these

(3) The length of longest path in a tree is called its :

(a) Centre (b) Radius

(c) Diameter (d) Walk

(4) For any connected graph with n vertices, e edges, its spanning tree has ____ chords.

(a) e – n + 1 (b) e – n – 1

(c) e + n + 1 (d) e – n + 2

(5) The number of common edges in circuit and cutset are :

(a) Even (b) Odd

(c) Empty (d) None of these

(6) If G is a planar graph with n vertices, e edges, f faces and k components then n – e + f = _____.

(a) k + 1 (b) k – 1

(c) n + k (d) n – k

(7) Subspaces W_ℓ and W_s are said to be orthogonal complements iff :

(a) $\dim(W_\ell \cup W_s) = 0$ (b) $\dim(W_\ell \cap W_s) = 0$

(c) $\dim(W_\ell \cup W_s) = 1$ (d) $\dim(W_\ell \cap W_s) = 1$

(8) Let Graph G be connected with 4 vertices and 5 edges then nullity =

(a) 9 (b) 5

(c) 2 (d) 4

(9) There is no rows with all zeros in the :

(a) Incidence matrix (b) Isolated matrix

(c) Cutset matrix (d) Path matrix

(10) If A(G) is an incidence matrix of a completed graph G with n vertices then rank of A(G) is :

(a) (n + 1)/2 (b) (n – 1)/2

(c) n – 1 (d) n + 1 1×10=10

UNIT—I

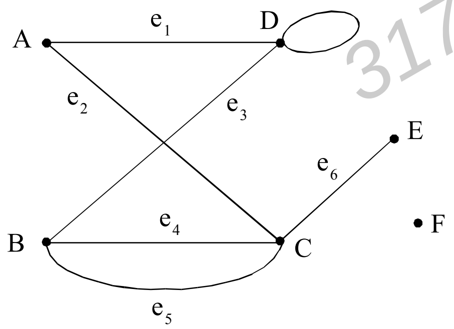
2. (a) Define (i) Regular graph, (ii) Null graph and show that the maximum number of edges in a simple graph of n vertices is n(n – 1)/2. 2+3

(b) Define complete graph and draw the graphs of the following chemical compounds :

(i) CH_4 (ii) C_2H_6

(iii) C_6H_6 (iv) N_2O_3 5

3. (p) From the graph given below answer the following :



(i) Write the degree of each vertex

(ii) Write odd degree vertices

(iii) Write adjacent vertices of vertex A

(iv) Is the graph simple ? Why ? 1+1+1+2

(q) Define :

(i) Path

(ii) Circuit

Let u and v be vertices in a graph G. If there are two different walks from u to v then prove that G contains a circuit. 1+1+3

UNIT—II

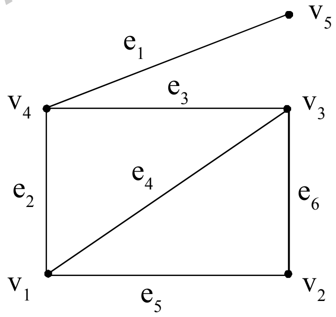
4. (a) Define centre of a tree and show that every tree has either one or two centres. 1+4

(b) If G is a graph with n vertices then prove that following statements are equivalent :

(i) G is a tree

(ii) G is connected and has n– 1 edges. 5

5. (p) Define spanning tree and find out all possible spanning trees of the following graph :



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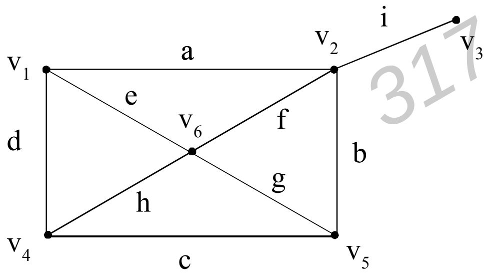
(q) Define Ecentricity of a vertex. Prove that the distance between two vertices in a connected graph is a metric. 1+4

UNIT—III

6. (a) Define Cutset. Find all the cutsets in the following graph :



1+4

(b) Prove that the ringsum of any two cutsets in a graph is either a third cutset or an edge disjoint union of cutsets. 5

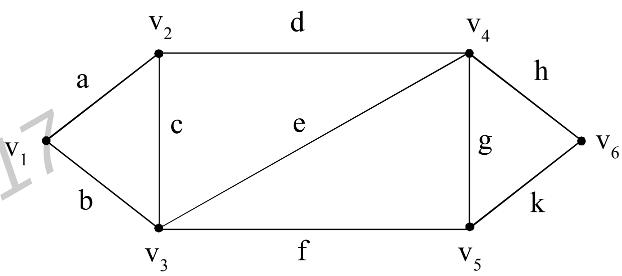
7. (p) Prove that every cutset in a connected graph G must contain at least one branch of every spanning tree of G. 5

(q) Define :

(i) Fundamental cutset

(ii) Fundamental circuit.

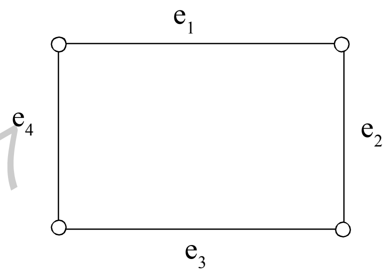
Find fundamental cutsets with reference to spanning tree : T = {b, c, e, h, k}



2+3

UNIT—IV

8. (a) Let G be a graph given as in figure. Find W_G, W_s, W_r, $W_r \cap W_s$, $W_r \cup W_s$ with spanning tree $T = \{e_1, e_2, e_3\}$.



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(b) Show that the set of all cutset vectors including zero vector in W_G forms a subspace of W_G. 5

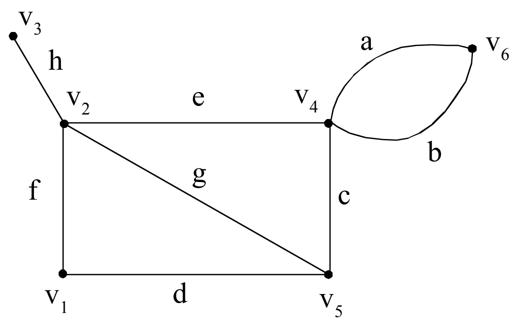
9. (p) Show that subspace W_r and W_s are orthogonal complements iff $W_r \cap W_s = 0$ i.e. $W_r \cap W_s = \{\phi\}$. 5

(q) Prove that the dot product of two vectors, one representing a subspace g_1 and other g_2 is zero if the number of edges common to g_1 and g_2 is even and the dot product is 1 if the number of common edges is odd. 5

UNIT—V

10. (a) Prove that the reduced incidence matrix of a graph is non-singular iff the graph is a tree. 5

(b) Define circuit matrix. Find the circuit matrix of the following graph :



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11. (p) Let A and B be respectively , the incidence matrix and the circuit matrix of a loop free graph whose columns are arranged using the same order of edges. Then show that every row of A is orthogonal to every row of B i.e $A \cdot B^T = 0$, $B \cdot A^T = 0 \pmod 2$.

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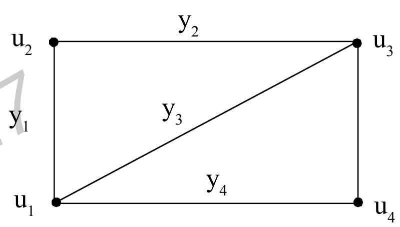
(q) Define cutset matrix. Find the cutset matrix of the following graph :



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