B.Sc. Part—I Semester—II Examination

MATHEMATICS

(Vector Analysis & Solid Geometry)

Paper—IV

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it at once only.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :

(1) The vectors $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar and $x\bar{a} + y\bar{b} + z\bar{c} = 0$ then :

(a) x + y = z
(b) x = y = z = 1
(c) x = y = z = 0
(d) x = y + z

(2) If two vectors in a scalar triple product are same then the scalar triple product is :

(a) One
(b) Two
(c) Three
(d) Zero

(3) The equation of osculating plane is :

(a) $(\bar{R} - \bar{r}) \cdot \bar{b} = 0$
(b) $(\bar{R} - \bar{r}) \cdot \bar{t} = 0$
(c) $(\bar{R} - \bar{r}) \cdot \bar{n} = 0$
(d) None of these

(4) A line perpendicular to both $\bar{t}$ and $\bar{n}$ is called :

(a) Tangent
(b) Binormal
(c) Principal normal
(d) None of these

(5) A vector $\bar{f}$ is solenoidal if :

(a) div $\bar{f} = 0$
(b) curl $\bar{f} = 0$
(c) div $\bar{f} \neq 0$
(d) curl $\bar{f} \neq 0$

(6) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then div r = _____.

(a) Zero
(b) One
(c) Three
(d) Two

(7) The curve of intersection of two spheres is a :

(a) Plane
(b) Circle
(c) Sphere
(d) Line

(8) If the radius of the circle is equal to the radius of the sphere, then the circle is :

(a) a small circle
(b) an imaginary circle
(c) great circle
(d) none of these

(9) The equation of the cone through the three axes is of the form :

(a) $fyz + gzx + hxy = 0$ (b) $fyz - gzx - hxy = 0$

(c) $fyz - gzx + hxy = 0$ (d) None of these

(10) The equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represent a cone if :

(a) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$ (b) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} < d$

(c) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} > d$ (d) None of these 10×1=10

UNTI—I

2. (a) Show that the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar only if $\bar{a} + \bar{b}, \bar{b} + \bar{c}, \bar{c} + \bar{a}$ are coplanar. 3

(b) If $\bar{a} = \bar{i} - \bar{j} - \bar{k},\ \bar{b} = \bar{j} + \bar{k},\ \bar{c} = \bar{i} - \bar{j},\ \bar{d} = \bar{i} + \bar{k}$ find :

(i) $(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d})$

(ii) $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})$. 2+2

(c) Prove that the necessary and sufficient condition for f(t) to have constant magnitude is $\bar{f} \circ \frac{d\bar{f}}{dt} = 0$. 3

3. (p) If $\bar{A} = x^2yz\,\bar{i} - 2xz^3\,\bar{j} + xz^2\,\bar{k}$ and $\bar{B} = 2z\,\bar{i} + y\,\bar{j} - x^2\,\bar{k}$ find $\frac{\partial^2}{\partial x \cdot \partial y}(\vec{A} \times \vec{B})$ at (1, 0, –2). 3

(q) Prove that :

$$\bar{i} \times (\bar{a} \times \bar{i}) + \bar{j} \times (\bar{a} \times \bar{j}) + \bar{k} \times (\bar{a} \times \bar{k}) = 2\bar{a}$$ 3

(r) Prove that :

(i) $\int \bar{f} \cdot \frac{d\bar{f}}{dt}\, dt = \frac{1}{2}\bar{f}^2 + c$

(ii) $\int \bar{f} \times \frac{d^2\bar{f}}{dt^2}\, dt = \bar{f} \times \frac{d\bar{f}}{dt} + \bar{c}$,

where $\bar{c}$ is constant. 2+2

UNIT—II

4. (a) State and prove Serret-Frenet formulae. 1+4

(b) Find the radii of curvature and torsion of the helix $x = a \cos \theta$, $y = a \sin \theta$, $z = a\theta \tan \alpha$, where α is constant. 5

5. (p) If the tangent and binormal at a point of a curve makes angles θ and ϕ respectively with fixed direction. Show that :

$$\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi} = -\frac{k}{\tau}$$ 5

(q) Prove that :

$$[\bar{b}', \bar{b}'', \bar{b}'''] = \tau^3(k'\tau - k\tau') = \tau^5\left(\frac{k}{\tau}\right)'$$ 5

UNIT—III

6. (a) Compute the line integral $\int_C y^2dx - x^2dy$, about the triangle whose vertices are (1, 0), (0, 1) and (–1, 0). 5

(b) Prove that $r^n\vec{r}$ is irrotational. Find the values of n when it is solenoidal. 3+2

7. (p) Find the constants a, b, c so that $\bar{f} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational. 5

(q) Let R be the closed bounded region in the xy-plane whose boundary is a simple closed curve C which may cuts by any line parallel to the coordinate axes in at most two points. Let M(x, y) and N(x, y) be functions that are continuous and have continuous partial derivatives $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$ in R. Then prove that :

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy = \int_C Mdx + Ndy$$ 5

UNIT—IV

8. (a) Find the equation of a sphere which passes through the origin and intercepts lengths a, b and c on the axes respectively. 5

(b) State and prove the condition for the orthogonality of two spheres. 1+4

9. (p) Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 5

(q) Find the equation of the sphere which passes through the points (1, –3, 4), (1, –5, 2) and (1, –3, 0) and whose centre lies on the plane $x + y + z = 0$. 5

UNIT—V

10. (a) Find the equation of a cone whose vertex is the point (α, β, γ) and whose guiding curve a conic in the xy-plane. 5

(b) Find the equation of right circular cone whose vertex is (2, –3, 5), axis makes equal angles with the coordinate axes and semi vertical angle is 30º. 5

11. (p) Prove that every homogeneous equation of second degree in x, y and z represents a cone whose vertex is at the origin. 5

(q) Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2}=\frac{y}{3}=\frac{z-3}{1}$. 5