AD-1798

B.Sc. (Part–I) Semester–II (OLD) Examination

MATHEMATICS

(Vector Analysis & Solid Geometry)

Paper - IV

Time : Three Hours] [Maximum Marks : 60

Note :– (1) Question No. **1** is compulsory. Attempt it once only.

(2) Attempt **one** question from each unit.

1. (1) Let $\bar{a}$ and $\bar{b}$ be any two non-zero vectors then $\bar{a}$ and $\bar{b}$ are parallel if :

(a) $\bar{a}+\bar{b}=1$ (b) $\bar{a}\times\bar{b}=0$

(c) $\bar{a}\,.\,\bar{b}=0$ (d) $\bar{a}\,.\,\bar{b}\,.\,1$ 1

(2) Let $\bar{a}$ and $\bar{b}$ be any two non-zero vectors then $\bar{a}$ and $\bar{b}$ are orthogonal iff :

(a) $\bar{a}+\bar{b}=1$ (b) $\bar{a}\times\bar{b}=0$

(c) $\bar{a}\,.\,\bar{b}=0$ (d) $\bar{a}\,.\,\bar{b}=1$ 1

(3) If $\bar{r}=\bar{r}(s)$ is equation of space curve then curvature is equal to :

(a) $\dfrac{|\dot{r}\ \ddot{r}\ \dddot{r}|}{|\dot{r}\times\ddot{r}|^2}$ (b) $\dfrac{r}{|\ \dot{r}\ |}$

(c) $\dfrac{\dot{r}\times\ddot{r}}{|\dot{r}\times\ddot{r}|}$ (d) $\dfrac{|\dot{r}\times\ddot{r}|}{|\dot{r}|^3}$ 1

(4) The Darboux vector $\bar{d}$ can be written as :

(a) $\bar{d}=y\bar{t}+x\bar{b}$ (b) $\bar{d}=y\bar{t}-x\bar{b}$

(c) $\bar{d}=y\bar{t}+\bar{b}$ (d) None 1

(5) If $\bar{f}$ and $\bar{g}$ are irrotational then .... is solenoidal.

(a) $\bar{f}\times\bar{g}$ (b) $\bar{f}\,.\,\bar{g}$

(c) $\bar{f}$? $\bar{g}$ (d) None 1

(6) A vector $\bar{f}$ is irrotational if :

(a) grad $\bar{f}=0$ (b) div $\bar{f}=0$

(c) curl $\bar{f}=0$ (d) None 1

(7) The curve of intersection of two spheres is a :

(a) Circle (b) Sphere

(c) Plane (d) Cone 1

(8) If a plane touches a sphere, the length of the perpendicular from the centre of the sphere is equal to its :

(a) Radius (b) Diameter

(c) Chord (d) None 1

(9) The equation of a cone with vertex at the origin is ....

(a) Homogeneous (b) Non-homogeneous

(c) Both (a) and (b) (d) None 1

(10) Every section of a right circular cone by a plane perpendicular to its axis is a :

(a) Sphere (b) Cone

(c) Cylinder (d) Circle 1

UNIT–I

2. (a) Prove that :

$$\bar{i}\times(\bar{a}\times\bar{i})+\bar{j}\times(\bar{a}\times\bar{j})+\bar{k}\times(\bar{a}\times\bar{k})=2\bar{a}.$$ 5

(b) Prove that :

$$(\bar{a}\times\bar{b}).(\bar{b}\times\bar{c})\times(\bar{c}\times\bar{a})=(\bar{a}\,\bar{b}\,\bar{c})^2.$$ 5

3. (p) If $\bar{f}$ and $\bar{g}$ are vector function of t, then :

$$\frac{d}{dt}(\bar{f}.\bar{g})=\bar{f}.\frac{d\bar{g}}{dt}+\frac{d\bar{f}}{dt}.\bar{g}.$$ 5

(q) If $\bar{r}(t)=5t^2.\bar{i}+t\bar{j}-t^3\bar{k}$

Prove that $\int_1^2 \bar{\varepsilon}\times\frac{d^2\bar{\varepsilon}}{dt^2}\,dt$ 5

$$=-14\bar{i}+75\bar{j}-15\bar{k}$$

UNIT–II

4. (a) Show that the tangent at any point on the curve whose equations are $x = 3u$, $y = 3u^2$, $z = 2u^3$ makes a constant angle with the line $y = z - x = 0$. 5

(b) Prove that the necessary and sufficient condition that a curve lies in a plane is $\tau = 0$. 5

5. (p) If the tangent and the binormal at a point of a curve make angles θ, ϕ respectively with a fixed direction then show that

$$\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi}=\frac{-k}{\tau}$$ 5

(q) Darboux vector $\bar{d}$ is constant iff k and τ are constants and that $\bar{d}$ has a fixed direction iff $\frac{k}{\tau}$ is constant. 5

UNIT–III

6. (a) Define directional derivative of ϕ and find the directional derivative of $\phi = xy^2 + yz^2$ at point (z, –1, 1) in the direction of the vector $\bar{i}+2\bar{j}+2\bar{x}$. 1+5

(b) If $\bar{v}=\bar{w}\times\bar{\varepsilon}$ then prove that $\bar{w}=\frac{1}{2}\text{curl}\,\bar{v}$ where $\bar{w}$ is a constant vector. 4

7. (p) Computer the line integral $\int_e y^2dx - x^2dy$, about the triangle whose vertices are (1, 0) (0, 1) and (–1,0). 5

(q) Find the total work done in moving a particle in a force field given by

$\overline{F} = 2xy\overline{i} + 3z\overline{j} - 6x\overline{k}$ along the curve :

$x = t^2 - 1, y = t, z = t^3$ from t = 0 to t = 1. 5

UNIT–IV

8. (a) Find the co-ordinates of the centre and the radius of the circle $x + 2y + 2z = 15, x^2 + y + z^2 - 2y - 4z = 11$ 5

(b) Show that the equation of the sphere through the four points (0, 0, 0) (–a, b, c), (a, –b, c), (a, b, –c) is $\frac{x^2+y^2+z^2}{a^2+b^2+c^2} - \frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$ and determine its radius. 5

9. (p) Find the equation of the sphere that passes through the circle :

$x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0, 3x - 4y + 5z - 15 = 0$ and cuts the sphere $x^2 + y^2 + z^2 + 2x + ay - 6z + 11 = 0$ orthogonally. 5

(q) Find the equation of the sphere which touches the sphere :

$x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at (1, 2, –2) and passes through the point (1, –1, 0). 5

UNIT–V

10. (a) Find the equation of the cone whose vertex is the point (1, 0, 1) and whose guiding curve is $z = 0, x^2 + y^2 = 4$. 5

(b) Show that the equation of a cone with vertex at the origin is homogenous. 5

11. (p) Find the equation of the right circular cylinder which passes through the circle :

$x^2 + y^2 + z^2 = 9, x - y + z = 3$. 5

(q) Find the equation of the right circular cylinder of radius z and whose axis is the line :

$\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$. 5