B.Sc. Part—II Semester—III Examination
MATHEMATICS
(Advanced Calculus)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory, attempt once.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :

(1) The limit of sequence if exist then sequence is :

(a) Convergent (b) Divergent

(c) Constant (d) None of these

(2) A sequence is decreasing if :

(a) $S_n \leq S_{n+1}$ (b) $S_n \geq S_{n+1}$

(c) $<S_n>$ (d) $S_n = S_{n+1}$

(3) In a P-series where P = 1 is called :

(a) Geometric series (b) P-series

(c) Harmonic series (d) Alternating series

(4) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is :

(a) Convergent (b) Divergent

(c) Test fails (d) None of these

(5) A function f is said to be continuous at $x = x_0$ if :

(a) $\lim_{x \to x_0} f(x) = f(x_0)$ (b) $\lim_{x \to x_0} f(x) \neq f(x_0)$

(c) $\lim_{x \to x_0} f(x) = 0$ (d) None of these

(6) The value of $\lim_{(x,y) \to (3,1)} \frac{\tan^{-1}(xy - 3)}{\sin^{-1}(4xy - 12)}$ is :

(a) $\frac{3}{2}$ (b) $\frac{4}{5}$

(c) $\frac{1}{4}$ (d) 0

(7) The function f(p) has an absolute maxima at point p_0 in D if :

(a) $f(p) \geq f(p_0); \forall p \in D$ (b) $f(p) \leq f(p_0); \forall p \in D$

(c) $f(p) = f(p_0); \forall p \in D$ (d) $f(p) \neq f(p_0); \forall p \in D$

(8) If xu = yz, yv = xz and xw = xy then $\dfrac{\partial(x, y, z)}{\partial(u, v, w)} =$ ______ .

(a) 1 (b) 0

(c) $\dfrac{3}{4}$ (d) $\dfrac{1}{4}$

(9) Find the value of $\int_0^{\pi/2}\int_0^{2a\cos\theta} r^2 \sin\theta \, dr \, d\theta$:

(a) $\dfrac{2a}{3}$ (b) $\dfrac{3a^2}{4}$

(c) $\dfrac{2a^3}{3}$ (d) $\dfrac{a}{3}$

(10) Find the $\iint_S r\hat{n}\, dS =$ ______ .

(a) $\dfrac{3\pi}{4}$ (b) $\dfrac{\pi}{4}$

(c) 3V (d) $\dfrac{4V}{3}$ 1×10=10

UNIT—I

2. (a) Find the limit of sequence $\lim\limits_{n\to\infty} \dfrac{2 + 3(10)^n}{3 + 4(10)^n}$. 3

(b) Prove that $\lim\limits_{n\to\infty} \dfrac{1^2 + 2^2 + ... n^2}{2n(n+1)(n+2)} = \dfrac{1}{6}$. 3

(c) Prove that every convergent sequence of real number is a Cauchy sequence. 4

OR

3. (p) The sequence $< S_n >$ is said to be convergent if and only if $\forall\ \varepsilon > 0$, $\exists$ +ve no. M such that $n, m \geq M \Rightarrow |S_m - S_n| < \varepsilon$. 5

(q) Show that the sequence defined $S_n = \dfrac{1}{3+1} + \dfrac{1}{3^2+1} ..... \dfrac{1}{3^n+1}$ is monotonic and bounded. 5

UNIT—II

4. (a) Prove that if geometric series $\sum_{n=1}^{\infty} x^{n-1}$ is convergent if x < 1 and divergent x ≥ 1. 5

(b) Test the convergence of the series $\sum_{n=1}^{\infty}\left[\left\{\frac{n+1}{n}\right\}^{n+1}-\left\{\frac{n+1}{n}\right\}\right]^{-n}$. 5

OR

5. (p) Define conditionally convergent with example. 2

(q) Test the convergence of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{1+x^n}$. 4

(r) Define :

(i) Abel's test

(ii) Dirichlet test. 4

UNIT—III

6. (a) Expand e^{xy} at the point (2, 1) upto first three term. 5

(b) Show that if $z = f(x, y) = \frac{y(x^2+y^2)}{y^2+(x^2+y^2)^2}$ then show that f has limit 0 as $(x, y) \to (0, 0)$ on a ray x = at, y = bt but f does not have limit 0 as $(x, y) \to (0, 0)$. 5

OR

7. (p) Prove that by using r – δ definition of a limit of function $\lim_{(x,y)\to(4,-1)}(3x-2y)=14$. 4

(q) Define Intermediate theorem. 2

(r) Expand $x^3 + y^3 - 3xy$ in powers of x – 2 and y – 3 i.e. at the point (2, 3). 4

UNIT—IV

8. (a) Locate the critical point and determine whether a local maximum occur for the function $f(x, y) = x^2 + 2xy + 3y^2 + 4x + 6y$ at these point. 5

(b) Using the method of Lagrange's Multipliers find the extreme values of f :

$$f(x, y) = \frac{x}{3} + \frac{y}{4} \text{ where } x^2 + y^2 = 1.$$ 5

OR

9. (p) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u, v)}{\partial(x, y)}$ if xy ≠ 1. State whether u and v are functionally related. Find the relationship. 5

(q) Find the maximum and minimum values of $x^3 + y^3 - 3axy$. 5

UNIT—V

10. (a) Evaluate the integral $\int_{-2}^{2} dy \int_{y^2-1}^{3} (x + 2y)\, dx$. 3

(b) Determine the limit of integration for the double integral $\iint_D f(x, y)\, dxdy$ for D is bounded by the parabolas $y^2 = x$ and $x^2 = y$. 4

(c) Evaluate the following integral by changing the order of integration $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dydx$.

3

OR

11. (p) Evaluate by changing the order of integration $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\, dydx}{\sqrt{x^2 + y^2}}$. 5

(q) Show that $\iint_S (ax\bar{i} + by\bar{j} + cz\bar{k})\,\bar{n}\, dS = \frac{4}{3}\pi\,(a + b + c)$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. 5