B.Sc. Part–II (Semester–III) Examination

MATHEMATICS

(Advanced Calculus)

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Q. No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each unit.

1. Choose the correct alternative :—

(1) A sequence $< s_n >$ is convergent if and only if _______.

(a) It is monotonic sequence (b) It is bounded sequence

(c) It is monotonic and bounded (d) It is monotonic and unbounded

(2) The geometric series $\sum ar^{n-1}$ is convergent if _______.

(a) $| r | < 1$ (b) $| r | > 1$

(c) $| r | = 1$ (d) $| r | = 0$

(3) A sequence is a function in which domain set is _______.

(a) Complex number (b) Real number

(c) Integers (d) Natural number

(4) The series $\sum \frac{1}{n^{3/2}}$ is _______.

(a) Convergent (b) Divergent

(c) Neither convergent nor divergent (d) None of these

(5) A function f(p) is said to have local minimum at p_o in D if it satisfies the condition _______.

(a) $f(p) \leq f(p_o) \ \forall \ p \in D$ (b) $f(p) \geq f(p_o) \ \forall \ p \in D$

(c) $f(p) \leq f(p_o) \ \forall \ p \in N\delta(p_o)$ (d) $f(p) \geq f(p_o) \ \forall \ p \in N\delta(p_o)$

(6) If $u = x + y$ and $v = x - y$ then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is _______.

(a) –1 (b) –2

(c) 1 (d) 2

(7) The value of $\lim_{x \to 2} \{ \lim_{y \to 1} (xy - 3x + 4) \}$ is ______.

(a) 0 (b) 1

(c) 2 (d) 3

(8) The necessary condition for maxima and minima of f(x, y) at the point (x_0, y_0) is :

(a) $f_x(x_0, y_0) \neq 0$ and $f_y(x_0 y_0) \neq 0$ (b) $f_x(x_0, y_0) \neq 0$ and $f_y(x_0, y_0) = 0$

(c) $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) \neq 0$ (d) $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$

(9) Stoke's theorem is the connection between :

(a) Line integral and Double integral (b) Line integral and Surface integral

(c) Volume integral and Surface integral (d) Volume integral and Double integral

(10) The value of $\int_0^1 \int_0^2 \int_0^3 dx\, dy\, dz$ is :

(a) 2 (b) 4

(c) 6 (d) 8 10×1=10

UNIT—I

2. (a) Prove that limit of sequence if it exists then it is unique. 4

(b) Show that $\lim_{n \to \infty} \left(\frac{n^n}{n!} \right)^{\frac{1}{n}} = e$. 3

(c) Prove that every convergent sequence of real numbers is a Cauchy sequence. 3

3. (p) Show that the sequence $< s_n >$; where $s_n = \left(1 + \frac{1}{n}\right)^n$ is convergent and its limit lies in between 2 and 3. 5

(q) Show that the sequence $< s_n >$ defined by $s_n = \frac{1}{3+1} + \frac{1}{3^2+1} + ... + \frac{1}{3^n+1}$ is monotonic and bounded. 5

UNIT—II

4. (a) Prove that the series $\sum \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \leq 1$. 5

(b) Test the convergence of the series $\sum \frac{n^3 + a}{2^n + a}$. 5

5. (p) Let $\sum x_n$ be a positive term series such that $\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = l$, then prove that the series $\sum x_n$ converges if $l < 1$ and diverges if $l > 1$. 5

(q) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ by using Cauchy's integral test. 5

UNIT—III

6. (a) Let real valued functions f and g be continuous in an open set $D \subseteq R^2$ then prove that $f - g$ is continuous in D. 4

(b) Let $z = f(x, y) = \frac{y(x^2+y^2)}{y^2+(x^2+y^2)^2}$; show that f has limit 0 as $(x, y) \to (0, 0)$ on the ray $x = at$; $y = bt$, but f does not have limit 0 as $(x, y) \to (0, 0)$. 3

(c) Obtain the expansion of $f(x, y) = x^2 - y^2 + 3xy$ at the point (1, 2). 3

7. (p) State and prove uniqueness of limit of function of two variables. 5

(q) Prove that the function $f(x, y) = x - y$ is continuous for all $(x, y) \in R^2$. 5

UNIT—IV

8. (a) Let f(x, y) be defined in an open region D and it has a local maximum or local minimum at (x_0, y_0). If the partial derivatives f_x and f_y exist at (x_0, y_0) then prove that $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$. 5

(b) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane $x - 2y + 2z = 9$. 5

9. (p) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction. 5

(q) If $xu = yz$, $yv = xz$ and $zw = xy$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 5

UNIT—V

10. (a) Evaluate by changing the order of integration $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dy.dx$. 5

(b) Evaluate $\iiint_V (2x+y)\,dx\,dy\,dz$; where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$; $z = 0$. 5

11. (p) Verify Gauss-Divergence theorem for

$$\bar{f} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$$ taken over

the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$; $0 \le z \le c$. 5

(q) Apply Stoke's theorem to evaluate

$$\oint_C [(x+y)dx + (2x-z)dy + (y+z)dz];$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0), (0, 0, 6). 5