B.Sc. Part–I (Semester–I) (CBCS) Examination

MATHEMATICS

(II) Differential and Integral Calculus

Time : Three Hours] [Maximum Marks : 60

Note :– (1) Q. No. **1** is compulsory.

(2) Attempt **one** question from each unit.

1. Choose the correct alternative : 10

(1) The function f(x) = tanx is discontinuous at :

(a) $x = \frac{\pi}{2}$ only

(b) $x = (2n+1)\frac{\pi}{2}, n = 0,1,2,...$

(c) $x = n\pi$ for all $n \in N$

(d) None of these

(2) The function $y^2 = x$ is :

(a) Single valued function

(b) Multiple valued function

(c) Even function

(d) None of these

(3) The value of $\lim_{x\to 0}(1+x)^{1/x}$ is :

(a) e

(b) e^2

(c) 0

(d) 1

(4) The function $f(x) = \begin{cases} \frac{\sin x}{x} ; & x \neq 0 \\ 1 ; & x = 0 \end{cases}$ has :

(a) Discontinuity at x = 0

(b) Removable discontinuity at x = 0

(c) Simple discontinuity at x = 0

(d) None of these

(5) If $y = a^x$ then y_n = ?

(a) $a.a^x$

(b) $a^x.(\log a)$

(c) $a^x.(\log a)^n$

(d) None of these

(6) An expression of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ is known as an :

(a) indeterminant form

(b) determinate form

(c) indeterminate form

(d) None of these

(7) For $f(x) = x^2$; $g(x) = x^3$ in [1, 3] then the value of 'c' by Cauchy's mean value theorem is :

(a) $\frac{6}{13}$ (b) $\frac{13}{6}$

(c) 0 (d) None of these

(8) Expansion of the function e^x is :

(a) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - .....$ (b) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ...$

(c) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .....$ (d) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

(9) If $I_n = \int \cos^n x dx$ then I_n = ...

(a) $-\frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$ (b) $\frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$

(c) $\frac{1}{n}\cos^{n+1} x \sin x + \frac{n+1}{n} I_{n+2}$ (d) None of these

(10) The value of $\int_0^a \frac{x^4 dx}{\sqrt{a^2 - x^2}}$ is :

(a) $\frac{3a^3\pi}{16}$ (b) $\frac{3a^2\pi}{16}$

(c) $\frac{3a^4\pi}{16}$ (d) None of these

UNIT–I

2. (a) Define limit of a function and if $\lim_{x \to x_0} f(x) = A$ and $\lim_{x \to x_0} g(x) = B$ then prove that $\lim_{x \to x_0} [f(x) + g(x)] = A + B$. 6

(b) If $f(x) = \frac{|x-2|}{x-2} + 2x + 3;\ x \neq 2$. Is the $\lim_{x \to 2} f(x)$ exist ? 4

OR

3. (c) Give E-S definition of limit. Using it 1

prove that $\lim_{x \to 2} (3x + 5) = 11$ 4

(d) If $\lim_{x \to x_0} f(x)$ exist, then it is unique. 5

UNIT–II

4. (a) Define uniform continuity and prove that any continuous function f defined on a closed interval [a, b] is uniformly continuous. 6

(b) Using E–S definition of continuity show that the function $f(x) = x^2$ is continuous for all real values of x. 4

OR

5. (c) If $f(x) = \begin{cases} \dfrac{\sin x}{x} \; ; \; x \neq 0 \\ 0 \; ; \; x = 0 \end{cases}$

show that f(x) has removable discontinuity at x = 0. 6

(d) Using E–S definition of continuity show that the function f(x) = 7x–3 is continuous at x = 3.4

UNIT–III

6. (a) If f(x) is differentiable at $x = x_0$ then prove that it is continuous at $x = x_0$. 4

(b) If $y = e^{ax} \cos(bx + c)$ then prove that $y_n = E^n e^{ax}. \cos(bx + c + n\theta)$ where $E = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\frac{b}{a}$. 6

OR

7. (c) If $y = \sin(m\sin^{-1}x)$ then prove that

(i) $(1 - x^2)y_2 - xy_1 + m^2y = 0$

(ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$. 5

(d) Evaluate :

(i) $\lim\limits_{x \to \infty} \dfrac{\log x}{x^n}$ 2

(ii) $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^2 - \tan x}$. 3

UNIT–IV

8. (a) State and prove Lagrange's mean value theorem. 6

(b) Verify Rolle's theorem for the function $f(x) = e^{-x} \sin x$ in $[0, \pi]$. 4

OR

9. (c) If f(x) and g(x) are two functions such that : 6

(i) f(x) and g(x) are continuous in [a, b]

(ii) f(x) and g(x) are differentiable in (a, b) and

(iii) $g'(x) \neq 0$ for $x \in (a, b)$.

Then there exist atleast one point $c \in (a, b)$ such that $\dfrac{f'(c)}{g'(c)} = \dfrac{f(b) - f(a)}{g(b) - g(a)}$.

(d) Obtain the Taylor's series expansion for the function $f(x) = x^4 + x - 2$ at a = 1. 4

UNIT–V

10. (a) Integrate $\int \frac{(x^2+2x+3)}{\sqrt{x^2+x+1}}dx$. 4

(b) If $I_n = \int \sec^n x dx$ then prove that $I_n = \frac{1}{n-1}\sec^{n-2} x \tan x + \frac{n-2}{n-1} I_{n-2}$. 6

OR

11. (c) If $I_n = \int \sec^n x dx$ then prove that $I_n = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}$. 6

(d) Show that $\int_0^{\pi/4} \sin^4 x dx = \frac{(3\pi - 8)}{32}$. 4