B.Sc. Part—I Semester—II Examination

MATHEMATICS

(Differential Equation : Ordinary & Partial)

Paper—III

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory. Solve it in one attempt only.

(2) Attempt one question from each Unit.

1. Choose the correct alternative (**1** mark each) :

(1) If the equation contains two arbitrary constants, then the resulting DE is of order :

(a) One (b) Two

(c) Three (d) Four

(2) The D.E. Mdx + Ndy = 0 is exact if :

(a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (b) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (d) None of these

(3) The particular integral of $f(D)y = e^{-ax}$ is :

(a) e^{ax} (b) $\frac{1}{f(a)}e^{-ax}$; $f(a) \neq 0$

(c) $\frac{1}{f(-a)}e^{ax}$; $f(-a) \neq 0$ (d) $\frac{1}{f(-a)}e^{-ax}$; $f(-a) \neq 0$

(4) The solution of DE $(D^2 - 4)Y = 0$ is :

(a) $c_1e^{2x} + c_2e^{-2x}$ (b) $(c_1 + c_2x)e^{2x}$

(c) A cos 2x + B sin 2x (d) None of these

(5) The particular integral of $Y'' + PY' + QY = 0$ is $y = e^x$ if :

(a) 1 – P + Q = 0 (b) 1 + P + Q = 0

(c) $m^2 + Pm + Q = 0$ (d) P + Q · x = 0

(6) The roots of the auxiliary equation of the DE $(D^2 + 5D + 6)Y = 0$ are :

(a) Real and distinct (b) Real and equal

(c) Complex (d) Real and Complex

(7) The PDE's f(x, y, p, q) = 0; g(x, y, p, q) = 0 are compactible if :

(a) $xJ_{xp} + yJ_{yq} = 0$ (b) $J_{xp} - J_{yq} = 0$

(c) $J_{xp} + J_{yq} = 0$ (d) $J_{xp} + J_{yq} + pJ_{zp} = 0$

(8) The general form of PDE is :

(a) f(x, y, z, p, q) = 0 (b) f(x, y, p, q) = 0

(c) f(x, y, z, p) = 0 (d) f(x, y, z) = 0

(9) The complete integral of f(x, p) = g(y, q) is :

(a) $z = \int h(x, a)\, dx$ (b) $z = \int k(y, a)\, dy$

(c) $z = \int h(x, a)\, dx + \int k(y, a)\, dy + b$ (d) None of these

(10) The solution of PDE $\gamma = a^2 t$ is :

(a) $z = F_1(y - ax) + F_2(y - ax)$ (b) $z = F_1(y - ax) + F_1(y - ax)$

(c) $z = F(y + ax)$ (d) $z = F_1(y + ax) + F_2(y - ax)$ 10

UNIT—I

2. (a) Solve the DE $(1 + y^2)dx = (\tan^{-1}y - x)dy$. 5

(b) Solve the DE $\cos x\, dy = y(\sin x - y)dx$. 5

3. (p) Solve the DE :

$$x\,dx + y\,dy = a^2 \frac{xdy - ydx}{x^2 + y^2}$$ 5

(q) Find the orthogonal trajectories of the cardiods $r = a(1 - \cos \theta)$, where 'a' is the parameter. 5

UNIT—II

4. (a) Solve the DE :

$$y'' - 4y' + 4y = e^{2x} + \sin 2x$$ 5

(b) Solve the DE :

$$(D^2 - 4D + 4)y = 8x^2e^{2x} \sin 2x$$ 5

5. (p) Solve the DE :

$$(x + a)^2 \frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$$ 5

(q) Solve the DE :

$$y'' + 3y' + 2y = 4x - 20 \cos 2x$$ 5

UNIT—III

6. (a) Solve the DE $(x \sin x + \cos x)\frac{d^2y}{dx^2} - x \cos x \frac{dy}{dx} + y \cos x = 0$ by changing the dependent variable. 5

   (b) Solve the DE $y'' - y = \frac{2}{1 + e^x}$ by variation of parameters. 5

7. (p) Solve $x^2y'' + xy' + 10y = 0$ by changing the independent variable from x to $z = \log x$. 5

   (q) Solve the simultaneous equations $D^2x - 2y = 0$ and $D^2y + 2x = 0$. 5

UNIT—IV

8. (a) Find the general integral of the PDE $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. 5

   (b) Solve the PDE $p^2 + q^2 = k^2$. 5

9. (p) Find the complete solution of $(x^2 + y^2)(p^2 + q^2) = 1$. 5

   (q) Find the complete integral of $z = p^2x + q^2y$. 5

UNIT—V

10. (a) Solve by Charpit's method :

    $pxy + pq + qy = yz$ 5

    (b) Find a surface passing through the two lines $z = x = 0$, $z - 1 = x - y = 0$, satisfying $r - 4s + 4t = 0$. 5

11. (p) Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solutions. 5

    (q) Solve $(D^2 + 3DD' + 2D'^2)z = x + y$. 5