B.Sc. Part—III (Semester—V) Examination

MATHEMATICS

(Mathematical Methods)

Paper—X

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory. Attempt it once.

(2) Solve **one** question from each unit.

1. Choose correct alternative :

(1) All roots of $P_n(x) = 0$ are _____.

(a) Equal (b) Distinct

(c) Complex (d) None

(2) The value of $\int_{-1}^{1} x^3 P_3(x)\, dx$ is _____

(a) $\frac{4}{35}$ (b) $\frac{35}{4}$

(c) $\frac{-4}{35}$ (d) $\frac{-35}{4}$

(3) The Eigne values of Strum-Liouville problem are ______.

(a) Complex (b) Equal

(c) Real (d) None

(4) If $J_p(x) = \sqrt{\frac{2}{\pi x}} \cos x$, then what is the value of P ?

(a) $-1/2$ (b) $1/2$

(c) $3/2$ (d) $-3/2$

(5) Every Fourier series is a _____.

(a) Power series (b) Exponential series

(c) Trigonometric series (d) None

(6) The fundamental period of cos x is _____ .

(a) 4π (b) π

(c) 3π (d) 2π

(7) If $L[f(t)] = F(s)$ then $L[f(at)]$ is _____

(a) $aF\left(\frac{s}{a}\right)$ (b) $\frac{1}{a}F\left(\frac{s}{a}\right)$

(c) $F\left(\frac{a}{s}\right)$ (d) $F\left(\frac{s}{a}\right)$

(8) If $L[f(t)] = \frac{1}{s^2}, s > 0$ then f (t) is _____

(a) t^n (b) t^2

(c) 1 (d) t

(9) If Fourier transform is linear, then $F_s\,[e^{-x} - 3e^{-3x}]$ is _____.

(a) $F_s\,[e^{-x}] - 3F_s\,[e^{-3x}]$ (b) $F_s\,[e^{-x}] - F_s\,[3e^{-3x}]$

(c) $F\,[e^{-x}] - 3F\,[e^{-3x}]$ (d) $F\,[e^{-x}] - F\,[3e^{-3x}]$

(10) The Fourier transform of $F[u_t(x, t)]$ is _____.

(a) $F[u]$ (b) $\frac{d}{dt}F[u]$

(c) $i\lambda F[u]$ (d) $\frac{d^2}{dt^2}F[u]$ 10×1=10

UNIT-I

2\. (A) Find $P_3(x)$ by Rodrigues formula and show that :

$$\int_{-1}^{1} x^3 P_3(x)\,dx = \frac{4}{35}.$$ 5

(B) Show that, $\int_{-1}^{1} [P_n(x)]^2 dx = \frac{2}{2n+1}$, if m = n. 5

3\. (P) Prove that :

$$(2n+1)\,x\,P_n = (n+1)\,P_{n-1} + n\,P_{n-1}.$$ 5

(Q) Show that $P_n(1) = 1$ & $P_n(-x) = (-1)^n P_n(x)$. Hence or otherwise deduce that $P_n(-1) = (-1)^n$. 5

UNIT-II

4. (A) Show that, $J_p(x) = (-2)^p x^p \dfrac{d^p}{d(x^2)^p} J_0(x)$. 5

(B) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. 5

5. (P) Prove that :

$$J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + ...) = 1.$$

Deduce that, $|J_0(x)| \le |$ and $|J_p(x)| \le 2^{-1/2}, p \ge 1$. 5

(Q) Show that, $J'_{1/2}(x) J_{-1/2}(x) - J'_{-1/2}(x) J_{1/2}(x) = \dfrac{2}{\pi x}$. 5

UNIT–III

6. (A) Find Fourier series in $(0, 2\pi)$ for the function $f(x) = x^2$. 5

(B) Obtain Fourier sine series for $f(x) = \pi x - x^2$, $0 \le x \le \pi$.

Show that, $\dfrac{\pi^3}{32} = \dfrac{1}{1^3} - \dfrac{1}{3^3} + \dfrac{1}{5^3} - \dfrac{1}{7^3} + ...$ 5

7. (P) Expand $f(x) = 2x - x^2$ in the range $(0, 3)$ as a Fourier series with periods 3. 5

(Q) Find the Fourier series for e^{ax}, $-L < x < L$. 5

UNIT–IV

8. (A) Find Laplace transform of $f(t) = |t-1| + |t+1|$, $t \ge 0$. 4

(B) Prove that $\displaystyle\int_0^\infty \frac{e^{-at} - e^{-bt}}{t}\, dt = \ell n \frac{b}{a}$. 3

(C) Find $L^{-1}\left[\dfrac{6s-4}{s^2 - 4s + 20}\right]$. 3

9. (P) State and prove first shifting theorem for Laplace transform. 4

(Q) Find Laplace transform of $\displaystyle\int_0^t e^u u^3 du$. 3

(R) Verify Convolution theorem for $f_1(t) = t$, $f_2(t) = \cosh t$. 3

UNIT–V

10. (A) Find the Fourier transform of $f(x) = \begin{cases} 0, & |x| < a \\ 1, & |x| > a \end{cases}$.

Hence evaluate $\int_{-\infty}^{\infty} \frac{\sin\lambda a . \cos\lambda x}{\lambda} d\lambda$. 5

(B) Find the Fourier sine and cosine transforms of the function $f(x) = ae^{-x} + be^{-2x}, x \geq 0$. 5

11. (P) Show that Fourier cosine transform of $f(x) = e^{-x^2}$ is $\frac{1}{\sqrt{2}} e^{-\lambda^2/4}$. 5

(Q) Find the finite Fourier sine and cosine transforms of $f(x) = e^{ax}$ in $(0, \ell)$. 5