B.Sc. Part–II (Semester–IV) Examination

MATHEMATICS

CLASSICAL MECHANICS

PAPER—VIII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **one** question from each unit.

1. Choose the correct alternative :

(1) A simple pendulum with a variable length has _____.

(a) One degree of freedom (b) Two degrees of freedom

(c) Three degrees of freedom (d) Four degrees of freedom 1

(2) The shortest distance between two points in space is _____.

(a) A circle (b) An ellipse

(c) A parabola (d) A straight line 1

(3) If a bead is sliding along the wire then the constraint is _____.

(a) Holonomic (b) Non-holonomic

(c) Superfluous (d) None of these 1

(4) In a central force field, the areal velocity is ____.

(a) Not constant (b) Not conserved

(c) Conserved (d) Zero 1

(5) If two curves are closed in the sense of K^{th} order proximity then they are closed in the sense of _____.

(a) Higher order proximity (b) $(k + 1)^{th}$ order proximity

(c) Smaller order proximity (d) Any order proximity 1

(6) For an inverse square law the Virial theorem reduces to _____.

(a) $2\overline{T} = -n\overline{V}$ (b) $2\overline{T} = n\overline{V}$

(c) $2\overline{T} = \overline{V}$ (d) $2\overline{T} = -\overline{V}$ 1

(7) A coordinate q_i is said to be cyclic if and only if _____.

(a) $\frac{\partial L}{\partial q_i} = 0$ (b) $\frac{\partial L}{\partial q_i} < 0$

(c) $\frac{\partial L}{\partial q_i} > 0$ (d) None of these 1

(8) The general displacement of a rigid body with _____ point fixed is a rotation about some axis.

(a) One (b) Two

(c) Three (d) Four 1

(9) A square matrix A is said to be orthogonal if _____.

(a) $A = A^T$ (b) $A^T = A^{-1}$

(c) $A = A^{-1}$ (d) None of these 1

(10) Consider the statements :

A : Finite rotations do not commute

B : Infinitesimal rotations commute

(Select the correct answer from the following) :

(a) A is true and B is false (b) A is false and B is true

(c) Both A and B are false (d) Both A and B are true 1

UNIT—I

2. (a) Prove that the Lagrange's equations of motion can be written in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial d_i} = Q_i'$ for a system which is partly conservative. The quantity L refers to the conservative part and Q' to the forces which are not conservative. 5

(b) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$ then show that the common acceleration of the particle is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$. 5

3. (p) If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that $L' = L + \frac{dF}{dt}$, $F = F(q_1, \ldots, q_n\ t)$ also satisfies Lagrange's equations, where F is any arbitrary but differentiable function of its argument. 5

(q) Find the Lagrangian for the system consisting of a simple pendulum of mass m_2 with mass m_1 at the point of support which can move on a horizontal line lying in the plane in which m_2 moves. 5

UNIT—II

4. (a) Prove that the problem of motion of two masses interacting only with one another always be reduced to problem of the motion of a single mass. 5

(b) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. 5

5. (p) State and prove 'Virial Theorem'. 5

(q) In a central force field, for a particle moving in a plane, prove that

$$t = \int_{r_0}^{r} \frac{dr}{f} \text{ and } \theta = \theta_0 + \frac{h}{m}\int_{r_0}^{r} \frac{dr}{fr^2} \text{ where } f = \sqrt{\frac{2}{m}\left(E - V - \frac{h^2}{2mr^2}\right)}.$$ 5

UNIT—III

6. (a) Solve the variational problem

$$\delta\int_{1}^{2}\left[x^2 y'^2 + 2y^2 + 2xy\right]dx$$

given $y(1) = y(2) = 0$. 5

(b) If x does not occur explicitly in F, then prove that $F_{y'} y' - F = \text{constant}$. 5

7. (p) Show that the geodesics on a right circular cylinder is a circular helix. 5

(q) Show that the functional

$I[Y(x)] = \int_{1}^{2}\left[2y(x) + y'(x)\right] dx$ defined in the space $C_1[0,1]$ is continuous on the function $Y_0(x) = x$ in the sense of first order proximity. 5

UNIT—IV

8. (a) Derive Hamilton's equations from variational principle. 5

(b) Discuss the Routh's procedure. 5

9. (p) Prove that :

(i) $\frac{dH}{dt} = \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$

(ii) If a generalized coordinate do not appear in H then prove that the corresponding conjugate momentum is conserved. 3+2

(q) Show that Hamilton's principle can be derived from D' Alembert's principle. 5

UNIT—V

10. (a) Define Infinitesimal rotation. Prove that if $A = I + \varepsilon$ then the inverse rotation matrix $A^{-1} = I - \varepsilon$. 5

(b) Describe the frame rotation and obtain the rotation matrix. 5

11. (p) Prove that the change in the components of a vector $\bar{r}$ under the infinitesimal transformation of the coordinate system can be expressed as $d\bar{r} = \bar{r} \times d\bar{u}$ where $d\bar{u} = (du_1, du_2, du_3)$ is a vector satisfying an infinitesimal rotation. 5

(q) Prove that the rotation matrix is orthogonal. 5