B.Sc. Part–II (Semester–IV) Examination

MATHEMATICS

(Classical Mechanics)

Paper—VIII

Time : Three Hours] [Maximum Marks : 60

Note :—(1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each unit.

1. Choose the correct alternative :

(1) The constraints on a bead on a uniformly rotating wire is a free space is _______.

(a) Rheonomous (b) Scleronomous

(c) Rheonomous and Scleronomous (d) None of these

(2) A particle is constrained to move along the inner surface of a fixed hemispherical bowl. The number of degrees of freedom of the particle is ________.

(a) One (b) Two

(c) Three (d) Four

(3) Each planet describes an ellipse having the sun in one of its _______.

(a) Radius (b) Eccentricity

(c) Foci (d) Centre

(4) The square of the periodic time of the planet is proportional to the cube of _______.

(a) Minor axis of its orbit (b) Semi major axis of its elliptic orbit

(c) Foci of its orbit (d) None of these

(5) If the function F does not contain the variable x and y explicitly, the extremals are all _______.

(a) Straight lines (b) Circle

(c) Ellipse (d) A hyperbola

(6) The geodesics on a right circular cylinder is a _______.

(a) Circular helix (b) Great circles

(c) Straight line (d) None of these

(7) A co-ordinate q_i is said to be cyclic if and only if _______.

(a) $\frac{\partial L}{\partial q_i} > 0$ (b) $\frac{\partial L}{r q_i} < 0$

(c) $\frac{\partial L}{r q_i} = 0$ (d) None of these

(8) In Δ-variation Hamiltonian H is _______.

(a) Varies (b) Constant

(c) Zero (d) None of these

(9) During the motion of rigid body if any straight line inside the body keeps the same direction, then the motion is _______.

(a) Rotation (b) General motion

(c) Translation (d) None of these

(10) The sum of finite rotations depends on the ______ of the rotations.

(a) Degree (b) Order

(c) Both degree and order (d) None of these 10×1=10

UNIT—I

2. (a) Construct a Lagrangian for spherical pendulum and then obtain the Lagrange's equation of motion. 5

(b) Show that the Lagrange's equation $\left(\frac{\partial T}{\partial \dot{q}_i}\right)^{\cdot} - \frac{\partial T}{\partial q_i} = Q'$ can also be written in the form

$$\frac{\partial \dot{T}}{\partial \dot{q}_i} - 2\frac{\partial T}{\partial q_i} = Q_i .$$ 5

3. (p) Discuss the motion of a particle in a plane by using polar co-ordinates. 5

(q) Two particles of masses m_1 and m_2, are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common acceleration of the particle is $\left\{\frac{(m_1 - m_2)}{(m_1 + m_2)}\right\} g$. 5

UNIT—II

4. (a) Prove that for a system moving in a finite region of space with finite velocity, the time average of KE is equal to the virial of system i.e. $\overline{T} = -\frac{1}{2}\overline{\Sigma \overline{F}_i \circ \overline{r}_i}$. 5

(b) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. 5

5. (p) Prove that in a central force field the angular momentum of a particle remains constant. Also prove that central force motion is a motion in a plane. 3+2

(q) For a central force field, show that Kepler's second law is a consequence of the conservation of angular momentum. 5

UNIT—III

6. (a) Show that the functional

$$I[y(x)] = \int_0^1 x^3 \sqrt{1+y^2(x)}\, dx$$

define on the set of function $y(x) \in C[0, 1]$ is continuous $y_0(x) = x^2$ in the sense of zero-order proximity. 5

(b) If x does not occur explicitly in F, then $f_{y'} y' - F = \text{constant}$. 5

7. (p) Find the extremals of the functional

$$I[y(n)] = \int_0^{\pi} (y'^2 - y^2 + 4y\cos x).\, dx\,,\ y(0) = 0,\ y(\pi) = 0.$$ 5

(q) Show that Euler's - Ostrogradsky equation for a functional

$I[z(x, y)] = \iint_D (1+p^2+q^2)^{1/2}\, dx\, dy$ is $(1 + q^2)r - 2pqs + (1 + p^2)t = 0$. 5

UNIT—IV

8. (a) Derive the Hamilton's canonical equations. 5

(b) Construct the Routhian in spherical polar co-ordinates for a particle moving in space under the action of a conservative force field. 5

9. (p) Define Hamiltonian H. Prove that cyclic co-ordinate will be absent in Hamiltonian. 1+4

(q) For a single particle system, the least action principle yields $\Delta\int \sqrt{2m(H-V)}\, ds = 0$ where $ds = |d\bar{r}|$. 5

UNIT—V

10. (a) Prove that rotation matrix A is orthogonal. 5

(b) Define infinitesimal rotation. Prove that Infinitesimal rotation matrix $\in$ is antisymmetric. 1+4

11. (p) Prove that if A is any 3 × 3 rotation matrix then A is orthogonal and |A| = 1. 5

(q) Prove that in a plane, a rotation of frame through an angle θ, followed by another rotation of the frame through an angle ϕ is equivalent to a single rotation through an angle $\theta + \phi$. 5