B.Sc. (Part-II) (Semester–III) (Old) Examination

MATHEMATICS

(Elementary Number Theory)

Paper–VI

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No.**1** is compulsory and attempt it once only.

(2) Solve one question from each unit.

1. Choose the correct alternative (1 mark each) : 10

(i) If a|b and 1<|a| <b then a is called ________.

(A) Proper divisor (B) Improper divisor

(C) G.C.D. (D) None of these

(ii) The product of any m consecutive integers is divisible by ________.

(A) (m–1) ! (B) m !

(C) (m+1) ! (D) None

(iii) If integer a and b are relatively prime them ________.

(A) (a,b) =0 (B) (a,b) >1

(C) (a,b) =1 (D) None

(iv) The integers of the form $2^{2^n}+1$ are called ________.

(A) Fermat Number (B) Perfect Number

(C) Both (D) None of these

(v) If a ≡ b (mod m) then ________.

(A) m|a+b (B) m|a–b

(C) m|ab (D) None

(vi) The linear congruence a*x*≡ b (mod m) has a unique solution modulo m, if ________.

(A) (a, m) =1 (B) (a,m) > 1

(C) (a, m) < 1 (D) None of these

(vii) If ϕ is Euler function then for n = 4 the value of ϕ (n) =________.

(A) 1 (B) 2

(C) 3 (D) 4

(viii) For n = 18 the value of arithmetic function T (n) = ________.

(A) 6 (B) 39

(C) 36 (D) 12

(ix) Let "a" be a primitive root of "m" then ________.

(A) $O_m(a) = f(m)$ (B) $O_m(a) = f(m+1)$

(C) $O_m(a) = f(a)$ (D) None of these

(x) If P is an odd prime with Primitive root a, then primitive root of P^2 is either ________.

(A) a or a-P (B) a or a+P

(C) a or aP (D) None of these

UNIT-I

2. (a) Let a and b be integers that are not both zero. Then prove that a and b are relatively prime iff there exist integers x and y such that $xa+yb=1$. 5

(b) Find the gcd of 275 and –200 and express it in the form $275x + (-200)y$. 5

3. (c) Prove that a|bc and (a,b) =1then a|c. 4

(d) Prove that (a, b+ka) = (a,b) where a,b, $k \in Z$. 3

(e) Prove that a necessary and sufficient condition for [a,b]=ab is (a,b)=1 where $a,b \in N$. 3

UNIT-II

4. (a) Prove that every positive integer greater than one has at least one prime divisor. 6

(b) Prove that $(a^2,b^2) = c^2$ if (a,b) =c. 4

5. (c) Prove that every positive integer a > 1 can be written uniquely as a product of primes apart from the order in which the factors occur i.e. $a = p_1 p_2 .... p_r$ all p_i being prime. 6

(d) Prove that if 2^m-1 is prime then m is also a prime. 4

UNIT-III

6. (a) Prove that congruence is an equivalence relation. 5

(b) Prove that if $a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$ then $a \equiv b \pmod{[m_1, m_2]}$. 5

7. (c) Show that the system of congruence $x \equiv a \pmod m$ and $x \equiv b \pmod n$ has a solution iff (m,n)| (a–b). 5

(d) Find the remainder obtained upon dividing the sum

1!+2!+3! + ...+ 1000!+1001! by 12. 5

UNIT-IV

8. (a) Solve the linear congruence $5x \equiv 3 \pmod{14}$ using Euler's theorem. 5

(b) Prove that Möbius μ-function is multiplicative. 5

9. (c) Prove that m be a positive integer and a be an integer with (a,m) =1 then $a^{\phi(m)} \equiv 1 \pmod{m}$. 5

(d) Prove that if f is a multiplicative function then the arithmetic function $F(n) = \sum_{d|n} f(d)$ is also multiplicative. 5

UNIT-V

10. (a) Prove that if the +ve integer m has a primitive root, then it has total of $\phi(\phi(m))$ in congruent roots. 5

(b) Prove that the congruence $x^2 \equiv a \pmod{p}$ has either no solution or exactly two in congruent solution modulo p. 5

11. (c) Solve the quadratic congruence $x^2+7x+10 \equiv 0 \pmod{11}$. 6

(d) Find the primitive root of p = 41. 4