B.Sc. Part—I (Semester—I) (CBCS) (New) Examination

MATHEMATICS

(I : Algebra & Trigonometry)

Paper : DSC-I

Time : Three Hours] [Maximum Marks : 60

Note :—Question No. **1** is compulsory, attempt once.

1. Choose the correct alternative :

(i) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, then $|A| =$ _____

(a) –7 (b) 7

(c) 1 (d) –1

(ii) If $A = \begin{bmatrix} 1 & 4 \\ -1 & 2 \end{bmatrix}$, then adj A = _____

(a) $\begin{bmatrix} 2 & -4 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 \\ 4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ -4 & 1 \end{bmatrix}$ (d) None of these

(iii) Every square matrix satisfies its own characteristics equation is the statement of ...

(a) De Moivre's theorem (b) Cayley Hamilton theorem

(c) Descartes's rule (d) None of these

(iv) If λ is a characteristic root of matrix A then the characteristic root of matrix KA is : ....

(a) λ^{-1} (b) λ^2

(c) $K\lambda$ (d) $K\lambda^2$

(v) Every equation of degree 3 has .....

(a) 4 roots (b) 2 roots

(c) 3 roots (d) less than 2 roots

(vi) A polynomial of degree zero is called as _____ polynomial.

(a) Monic (b) Constant

(c) Quadratic (d) Linear

(vii) The complex conjugate of 2 + i is _____

(a) – 2 – i (b) – 2 + i

(c) 2 – i (d) None of these

(viii) The value of $e^{\frac{-\pi}{2}i}$ is _____

(a) – i (b) 1 + i

(c) 1 – i (d) 0

(ix) Which of the following is the expansion of e^x function ?

(a) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$ (b) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+...$

(c) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-...$ (d) None of these

(x) The value of $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$ is _____

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) π (d) 2π 10

UNIT-I

2. (a) Find inverse of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{bmatrix}$ by Adjoint method. 6

OR

(b) Find non-singular matrices P & Q s.t. PAQ is the normal form where matrix $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & -1 & -1\end{bmatrix}$. 6

(c) If A & B are two non-singular matrices of order n then prove that $(AB)^{-1} = B^{-1}A^{-1}$. 4

OR

(d) Find the inverse of matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by using column transformation. 4

UNIT-II

3. (a) Find the rank of matrix A by reducing it to its normal form : $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. 6

OR

(b) Verify Cayley - Hamilton theorem for matrix A & find A^{-1} if $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. 6

(c) Show that the eigen values of a Hermitian matrix are all real. 4

OR

(d) Show that eigen values of an idempotent matrix are either zero or unity. 4

UNIT-III

4. (a) Solve by using Cardano's method $x^3 - 15x = 126$. 6

OR

(b) Find the condition that the roots of the equation $x^3 - ax^2 + bx - c = 0$ are in

(i) Arithmetic progression

(ii) Geometric progression. 6

(c) Solve $x^4 - 2x^3 - 22x^2 + 62x - 15 = 0$ given that $2+\sqrt{3}$ is one root. 4

OR

(d) Show that the equation $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four complex roots. 4

UNIT-IV

5. (a) State & prove De-Moivre's theorem for positive & negative integers. 6

OR

(b) If sin (A+iB) = x + iy then show that :

(i) $\dfrac{x^2}{\cosh^2 B}+\dfrac{y^2}{\sinh^2 B}=1$

(ii) $\dfrac{x^2}{\sin^2 A}-\dfrac{y^2}{\cos^2 A}=1$. 6

(c) Show that the continue product of four values of $\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)^{3/4}$ is 1. 4

OR

(d) Show that $\tanh^{-1} x=\sinh^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$. 4

UNIT-V

6. (a) Find the sum of the series $\sin\alpha-\dfrac{\sin 3\alpha}{3!}+\dfrac{\sin 5\alpha}{5!}+\ldots$. 6

OR

(b) If $\dfrac{-\pi}{4}\le x\le\dfrac{\pi}{4}$ then show that $x=\tan x=\dfrac{-\tan^3 x}{3}+\dfrac{\tan^5 x}{5}-\ldots$. 6

(c) Prove that $4\tan^{-1}\dfrac{1}{5}-\tan^{-1}\dfrac{1}{70}+\tan^{-1}\dfrac{1}{99}=\dfrac{\pi}{4}$. 4

OR

(d) Show that $\log(\tan^{-1} x)-\log x=-\dfrac{1}{3}x^2+\dfrac{13}{90}x^4+\ldots$ where x lies between 0 & 1. 4