B.Sc. Part—II Semester—III (Old) Examination

MATHEMATICS

(Advanced Calculus)

Paper—V

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternatives :

(1) The value of $\lim\limits_{n \to \infty} \dfrac{4 + 3.10^n}{5 + 3.10^n}$ is :

(a) 4/5 (b) 0

(c) 1 (d) 4

(2) The sequence $< s_n >$, where $s_n = \dfrac{\sqrt{n}}{n+1}$ is :

(a) Monotonic increasing (b) Monotonic decreasing

(c) Constant sequence (d) Oscillatory sequence

(3) The Geometric Series $\sum\limits_{n=1}^{\infty} r^{n-1}$ is convergent if :

(a) $r = 0$ (b) $r = 1$

(c) $r < 1$ (d) $r > 1$

(4) The p-series $\sum \dfrac{1}{n^p}$ in which $p = 1$ then is called as :

(a) Geometric series (b) Exponential series

(c) Logarithmic series (d) Harmonic series

(5) If iterated limit of a function are not equal at point then :

(a) Limit exist at point (b) Limit does not exist

(c) Limit is zero (d) None of these

(6) If $\lim\limits_{(x, y) \to (x_0, y_0)} f(x, y) \neq f(x_0, y_0)$, then :

(a) f is continuous (b) f is continuous at (x_0, y_0)

(c) f is discontinuous (d) None of these

(7) If u = 2x – y, v = x + 4y, then $J = \frac{\partial(u, v)}{\partial(x, y)} =$ ______.

(a) $\frac{1}{9}$ (b) 9

(c) 1 (d) 10

(8) If J is the Jacobian of x and y w.r.t. u and v and J′ Jacobian of u, v w.r.t. x and y. Then J′ is :

(a) $\frac{1}{J}$ (b) J

(c) – J (d) J^2

(9) $\int_0^1 \int_1^3 xy^2 \, dydx =$

(a) $\frac{12}{3}$ (b) $\frac{13}{3}$

(c) $\frac{14}{3}$ (d) $\frac{15}{3}$

(10) The value $\int_0^2 \int_0^2 \int_0^2 dx\,dydz$ is :

(a) 6 (b) 8

(c) 4 (d) 2 1×10=10

UNIT—I

2. (a) Prove that every Cauchy sequence is bounded. 4

(b) Show that the sequence $< s_n > = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$ is convergent. 3

(c) Evaluate $\lim_{n \to \infty} \frac{1 + 2 + 3 + 4 + ... + n}{n^2}$. 3

OR

3. (p) A real sequence $<s_n>$ converges iff for each $\in > 0$, $\exists$ M $\in$ N such that $|s_m - s_n| < \in \quad \forall\, m, n \geq M$. Prove this. 4

(q) Prove that every convergent sequence of real numbers is a Cauchy sequence. 3

(r) Prove that limit of sequence if it exists is unique. 3

UNIT—II

4. (a) Test the convergence of series $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)}$ by using integral test. 4

(b) If the series Σx_n is convergent then prove that $\lim_{n \to \infty} x_n = 0$. 3

(c) Define :

(i) Harmonic series

(ii) Absolutely convergent

(iii) Conditionally convergent. 3

OR

5. (p) Prove that $\sum \frac{1}{n^p}$ converges when $p > 1$ and diverges when $p \le 1$. 4

(q) Test the converges of series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4}$ 3

(r) Test the converges of $\sum \left(\frac{n}{n+1}\right)^{n^2}$. 3

UNIT—III

6. (a) Prove that if limit of function f(x, y) at (x_0, y_0) exist then it is unique. 4

(b) Expand e^{xy} at (2, 1) upto 1st three terms. 3

(c) Using $\in - \delta$ $\det^n$ of limit of function prove that $\lim_{(x,y) \to (1,1)} (x^2 + 2y) = 3$. 3

OR

7. (p) Prove that if x, y are differentiable functions of u, v and u, v are differentiable functions of r, s, then prove that :

$$\frac{\partial(x, y)}{\partial(u, v)} \times \frac{\partial(u, v)}{\partial(r, s)} = \frac{\partial(x, y)}{\partial(r, s)}$$ 4

(q) Obtain the expansion of $f(x, y) = x^2 - y^2 + 3xy$ at point (1, 2). 3

(r) Prove that the function f(x, y) = x + y is continuous function $\forall\, x, y \in R^2$. 3

UNIT—IV

8. (a) If $x = r \cos \theta$, $y = r \sin \theta$ find $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$ and also prove that $JJ' = 1$. 4

(b) Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither maxima or minima at $(0, 0)$. 3

(c) Find the extremum of $\sin A \cdot \sin B \cdot \sin C$ subject to condition $A + B + C = \pi$. 3

OR

9. (p) Find by using Lagrange's method of multiplier the least distance of the origin from plane $x - 2y + 2z = 9$. 4

(q) If $xu = yz$, $yv = xz$, $zw = xy$, find $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$. 3

(r) Find the Maxima and Minima value of function $x^3 + y^3 - 3axy$. 3

UNIT—V

10. (a) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} x^2 \, dydx$. 5

(b) Evaluate by Stoke's theorem $\int_C (e^x dx + 2y dy - dz)$, where 'C' is the curve $x^2 + y^2 = 4$, $z = 2$. 5

OR

11. (p) Evaluate by changing the order of integration :

$$\int_0^{2a} \int_{x^2/4a}^{3a - x} f(x, y) \, dx \cdot dy$$ 5

(q) Evaluate :

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dxdydz$$ 5